A Novel Quadratically Constrained Quadratic Programming Method for Optimal Coordination of Directional Overcurrent Relays

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Abstract—The coordination of directional overcurrent relays (DOCRs) in meshed power grids with multiple sources is a constrained optimization problem, which has been stated in recent literature as linear (LP), nonlinear (NLP), and mixed integer nonlinear programming (MINLP) problem. In this paper, the DOCR coordination NLP problem is reformulated as an equivalent quadratically constrained quadratic programming (QCQP) model, leading to significant reduction of problem complexity. Another contribution of this work is the systematic problem statement using graph theory concepts. The proposed method is applied to three different meshed power systems, employing state-of-the-art optimization software. Simulation results demonstrate the efficacy and superiority of the proposed QCQP model over the prevailing NLP approach.

Index Terms—Directional overcurrent relays, nonlinear programming, pickup current setting, power system protection, quadratically constrained quadratic programming, relay coordination, time dial setting.

I. INTRODUCTION

PROTECTION schemes are responsible for the secure and incessant operation of power delivery systems, ensuring the fast and accurate isolation of the trouble area, when abnormal and adverse conditions occur [1]. Nowadays, power system protection is deemed to be one of the most critical concerns for electrical engineers and researchers, owing to the new operational frameworks imposed by the global demand for more “green” power generation. Especially, the constantly increasing penetration of distributed generation (DG) in modern power grids, as well as the adoption of new sophisticated configurations (e.g., microgrid architecture), has raised new challenging protection issues necessitating the implementation of advanced protection solutions.

In accordance with long-established protection practices, protection schemes composed of directional overcurrent relays are broadly used as secondary protection of transmission systems, cooperating with distance relays, and as the first line of defense in subtransmission and distribution networks. The dependability and reliability of power utilities depend upon the proper setting of primary and backup protective means. More specifically, backup protective devices should be asserted and initiate the tripping mechanism of the corresponding circuit breaker solely in case of primary relay mal-operation or breaker failure incident, and after a certain delay widely known as coordination time interval (CTI).

A. Literature Review

The aim of the generic DOCR coordination problem is to find proper values for time dial and pickup current settings, satisfying the so-called coordination constraints. Excluding the conventional methods based on expert rules, which were applied so far to power utilities and industrial networks, the coordination techniques could be categorized into three main classes: topological, analytic and optimization techniques.

The first category is characterized by the topological analysis of the studied power systems, including graph theory and functional dependencies [2]–[7]. In these works, the concept of minimum break point set (MBPS) was extensively described and implemented, in order to identify all possible simple loops of the examined systems, and then achieve coordination for all primary and backup relay pairs. Additionally, in [4], the notion of functional dependency between backup and primary relays was presented, transforming the coordination constraints into a set of functional dependencies similar to database systems.

As far as analytic techniques are concerned, several iterative numerical algorithms have been proposed [8]–[13]. In these approaches, the coordination constraints form a system of inequalities, and acceptable values of time dial and pickup current settings are calculated in each iteration. The computation process stops when the value of the objective function does not decrease between two consecutive iterations, or the relative error is below a predefined value.

Last but not least, various optimization methods have been proposed in literature for optimal coordination of DOCRs [14]–[30]. The DOCR coordination problem is a highly constrained optimization problem, which has been formulated and solved successfully as linear programming (LP), nonlinear programming (NLP), and mixed-integer nonlinear programming (MINLP) problem. In LP method [14]–[17], only time dial settings are considered as design variables, whereas pickup current settings have fixed values between

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acceptable overload and minimum fault currents. In [18], the coordination problem is formulated as an interval linear programming problem (ILP), considering uncertainty in network topology. Plenty of heuristic and nonlinear optimization methods have been proposed in [19]–[28] for solving the NLP form of the DOCR coordination problem, where both pickup current and time dial settings are assumed to be continuous variables. This assumption agrees with the specifications of modern digital and numerical relays, whose setting values have accuracy of many decimal places. The MINLP model considers relays with fixed taps (e.g., electromechanical relays), and therefore the pickup current settings are regarded as discrete design variables, in contrast to time dial settings which remain continuous [29], [30]. It should be noted that optimization techniques do not require the determination of break points to calculate optimal DOCR settings, since they consider inherently all possible primary-backup relay pairs.

The major contribution of this paper is the novel transformation of the prevailing NLP problem of numerical DOCRs coordination into an equivalent, but considerably simplified, minimization problem with linear objective function subject to linear and quadratic constraints. Actually, the relay coordination problem is formulated as a quadratically constrained quadratic programming (QCQP) problem, which reduces the computational effort dominated by function and derivative evaluations, enhancing thus the efficiency and robustness of the employed optimization software.

B. Paper Structure

The rest of the paper is outlined as follows. In Section II, a novel reformulation of the DOCR coordination problem is presented. The advantages of the proposed formulation over the existing NLP model are thoroughly discussed in Section III. Furthermore, we emphasize the capability of the scip solver in finding the global optimum of the QCQP formulated coordination problem. Detailed results obtained by the application of the proposed QCQP technique and the existing NLP approach to different test systems are demonstrated and juxtaposed in Section IV. Finally, in Section V, the conclusions of this work are highlighted.

II. PROPOSED FORMULATION OF THE DOCR COORDINATION PROBLEM

The coordination of DOCRs in a meshed and multi-source power system can be generally stated as an optimization problem, where the objective function to be minimized is the sum of operating times of each primary-backup (p/b) relay pair [31], [32] subject to specific constraints. The vast majority of methods in recent literature consider the sum of primary relays operating times as the objective function. However, the proposed formulation optimizes DOCRs overall operating performance, taking into account their response not merely when acting as primary, but also when acting as backup protective devices.

Furthermore, graph theory is applied to model the DOCR coordination problem in a compact form. The proposed formulation aims at the automatic determination of p/b relay pairs, based on a simple analysis of the network topology. As a result, there is no need to define the DOCR pairs from scratch for each case study, greatly reducing the computation effort.

A. Key Definitions

The aim of this section is to compile some essential definitions obtained from graph theory, which will be used in the mathematical statement of the DOCR coordination problem.

Let a power system consist of \( N = |\mathcal{N}| \) buses and \( M = |\mathcal{M}| \) branches, where \( \mathcal{N} \) and \( \mathcal{M} \) are the sets of buses and branches, respectively. The single line diagram of the system can be represented by a (undirected) graph \( \mathcal{G}(\mathcal{V}, \mathcal{E}) \), comprising of a set \( \mathcal{V}(\mathcal{G}) \) of vertices (corresponding to buses) with cardinality \( N = |\mathcal{V}(\mathcal{G})| \) and a set \( \mathcal{E}(\mathcal{G}) \subseteq \mathcal{V}(\mathcal{G}) \times \mathcal{V}(\mathcal{G}) \) of edges (corresponding to branches) with cardinality \( M = |\mathcal{E}(\mathcal{G})| \). An edge of \( \mathcal{G} \) that joins vertices \( i \) and \( j \) is denoted as \((i, j)\) or \( ij \). Two vertices \( i \) and \( j \) are called adjacent (or neighbors) when \((i, j) \in \mathcal{G}\).

The adjacency matrix \( A_G = [a_{ij}] \in \mathbb{R}^{N \times N} \) of a graph \( \mathcal{G} \) is defined by:
\[
a_{ij} = \begin{cases} 1, & \text{if } (i, j) \in \mathcal{E}(\mathcal{G}) \\ 0, & \text{otherwise} \end{cases}
\]

The set of neighbors of a vertex \( i \) is defined as \( \mathcal{N}_i = \{ j \in \mathcal{V} : (i, j) \in \mathcal{E} \} \). The degree (or, valency) of a vertex \( i \in \mathcal{G} \), denoted by \( d(i) \), is equal to the number of its neighbors \( \mathcal{N}_i \); i.e., \( d(i) = |\mathcal{N}_i| = \sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N} a_{ji} \).

The associated digraph or directed graph \( \mathcal{D}(\mathcal{G}) \) of \( \mathcal{G} \), consisting of a set \( \mathcal{V}(\mathcal{G}) \) of vertices and a set \( \mathcal{A}(\mathcal{D}) \) of arcs (directed edges), is derived from graph \( \mathcal{G} \) by replacing each of its edges by two oppositely oriented arcs with the same ends [33]. It is actually the complete biorientation of \( \mathcal{G} \), such that \( ij \in \mathcal{A}(\mathcal{D}) \) implies \( ji \in \mathcal{A}(\mathcal{D}) \).

Since relays are installed at both end buses \( i \) and \( j \) of a branch (edge) \((i, j) \in \mathcal{D}(\mathcal{G})\), the relays \( ij \) and \( ji \) are distinct, and thus, each relay can be considered as an arc of the digraph \( \mathcal{D}(\mathcal{G}) \). Therefore, the set \( \mathcal{R} \) of relays will be:
\[
\mathcal{R} = \mathcal{A}(\mathcal{D})
\]

The total number, \( \mathcal{N}_R \), of relays (cardinality of set \( \mathcal{R} \) ) is given by:
\[
\mathcal{N}_R = |\mathcal{R}| = 2M
\]

For each relay \( ij \), the following sets are defined:
\[
\mathcal{B}_ij = \mathcal{N}_i - \{ j \}, \forall ij \in \mathcal{R}
\]
where, constants $A$, $B$, $C$ ($C = 0$, for IEC curves) define the type of the characteristic (e.g. Standard Inverse, Very Inverse), $TD_{ij}$ is the time dial setting of relay $ij$, and $M_{ij,f}$ is the ratio of the short-circuit current $I_{f,j}$, seen by relay $ij$, for a fault at location $f$, to the pickup current setting $I_{pu,ij}$ of relay $ij$.

2) Bounds of design variables: Design variables of the NLP coordination method are the pickup current $I_{pu,ij}$ and time dial $TD_{ij}$ settings of each DOCR $ij$. Boundary values for pickup current settings can be defined taking into account the highest load and the lowest short-circuit current values, to avoid operation of relays under normal overload conditions, while ensuring their sensitivity to minimum faults. Concerning TD settings, relay manufacturers specify the acceptable lower and upper values:

$$I_{pu,ij}^{\min} \leq I_{pu,ij} \leq I_{pu,ij}^{\max}, \forall ij \in \mathcal{R} \quad (9)$$

$$TD_{ij}^{\min} \leq TD_{ij} \leq TD_{ij}^{\max}, \forall ij \in \mathcal{R} \quad (10)$$

3) Nonlinear bounds of relays operating time: Relays need a certain minimum amount of time to initiate the tripping mechanism of their corresponding circuit breaker, and should not be permitted to take long time to be asserted. Thus, bounds for relays operating time, either acting as primary or as backup protective mean, are defined below in accordance with the protection scheme requirements:

$$t_{ij,f}^{\min} \leq t_{ij,f} \leq t_{ij,f}^{\max}, \forall ij \in \mathcal{R}, \forall k \in B_{ij}, f = 1, \ldots, N_F \quad (11)$$

4) Nonlinear coordination inequality constraints: Short-circuit currents are sensed by both primary and backup relays; therefore the coordination of protective schemes has a direct influence on the reliability of power systems. The overcurrent elements of backup relays must be asserted only in case of their primary relay mal-operation and after a certain time gap, known as coordination time interval and commonly abbreviated to $CTI$. Typical values for electromechanical relays are 0.3 to 0.4 s, while reasonable $CTI$ values for numerical relays are 0.1 to 0.2 s. The previous description is made clear by the following inequality:

$$t_{ij,f}^{\max} - t_{ij,f} \geq CTI, \forall ij \in \mathcal{R}, \forall k \in B_{ij}, f = 1, \ldots, N \quad (12)$$

Similar constraints can be added in the DOCR coordination problem to take into consideration different topologies or potential contingencies (e.g. single line outages) for the examined power system, as has been described in [18], [29]. More specifically, a different set of coordination constraints for each considered scenario, containing all the derived p/b DOCR pairs, is added to the optimization model.

C. Proposed QCQP Formulation of the Coordination Problem

In this section, the proposed formulation of the DOCR coordination problem is described in detail. Compared to the NLP formulation, the QCQP approach introduces a new set of
variables which simplify considerably the functional expressions of both the objective function and the constraints, while maintaining mathematical equivalence to NLP.

It is assumed that primary and backup relay operating times are independent quantities. In addition, from (8) we derive,

\[
(t_{ij,f} - TD_y \cdot C) \left( M_{ij,f}^B - 1 \right) = TD_y \cdot A
\]

By introducing the following auxiliary variables,

\[
y_{ij,f} = t_{ij,f} - TD_y \cdot C
\]

\[
z_{ij,f} = M_{ij,f}^B
\]

\[
w_{ij} = TD_y \cdot A
\]

equality (13) becomes as follows:

\[
y_{ij,f}(z_{ij,f} - 1) = w_{ij}.
\]

In the sequel, variables \(z_{ij,f}\) are eliminated from the objective function and the constraints. Note that (16) is also valid for backup relays.

Combining (14) and (16), the operating times of all primary and backup relays, that is \(\forall ij \in \mathcal{R}, \forall k \in \mathcal{B}_i, f=1,\ldots,N_F\), are given by the following expressions:

\[
t_{ij,f} = y_{ij,f} + \frac{C}{A}w_{ij},
\]

\[
t_{ij,k}^f = y_{ij,k}^f + \frac{C}{A}w_{ij}^f.
\]

1) Linear Objective function: Substituting (18) and (19) into (7), the objective function is transformed into an equivalent linear one in terms of \(w_{ij}, y_{ij,f}, y_{ij,k}^f\), as follows:

\[
\min f = \sum_{f=1}^{N_F} \sum_{j \in \mathcal{R}} \left( y_{ij,f} + \frac{C}{A}w_{ij} \right) + \sum_{k \in \mathcal{B}_j} \left( y_{ij,k}^f + \frac{C}{A}w_{ij}^f \right) = \sum_{f=1}^{N_F} \sum_{j \in \mathcal{R}} \left( y_{ij,f} + \frac{C}{A}w_{ij} \right) + \sum_{k \in \mathcal{B}_j} \left( y_{ij,k}^f + \frac{C}{A}w_{ij}^f \right)
\]

The variable \(w_{ij}^f\), which corresponds to the TD value of relay \(ki\) considered as backup to relay \(ij\), is equal to \(w_{ki}\), since relay settings do not change when acting either as primary or backup protection. As a consequence:

\[
w_{ij} = w_{ij}^f, \forall ij \in \mathcal{R}, \forall l \in \mathcal{C}_{ij}.
\]

Observing the objective function in (20), it is evident that \(w_{ij}, y_{ij,f}, y_{ij,k}^f, \forall ij \in \mathcal{R}, \forall k \in \mathcal{B}_j, f=1,\ldots,N_F\) are the design variables of the new optimization problem.

2) Bounds of design variables: According to (10) and (16), the bounds of the \(w_{ij}\) design variables are:

\[
w_{ij}^{\min} \leq w_{ij} \leq w_{ij}^{\max}, \forall ij \in \mathcal{R}
\]

where, \(w_{ij}^{\min} = TD_y^{\min} A\), and \(w_{ij}^{\max} = TD_y^{\max} A\). Additionally, substituting (18) and (19) into (11), we obtain:

\[
t_{ij,f}^{\min} \leq y_{ij,f} \leq \frac{C}{A}w_{ij} \leq t_{ij,f}^{\max}
\]

\[
t_{ij,k}^{\min} \leq y_{ij,k}^f \leq \frac{C}{A}w_{ij}^f \leq t_{ij,k}^{\max}
\]

\[\forall ij \in \mathcal{R}, \forall k \in \mathcal{B}_j, f=1,\ldots,N_F.
\]

3) Linear inequality constraints: The new linear inequality coordination constraints are derived by substituting (18) and (19) into (12):

\[
y_{ij,f} + \frac{C}{A}w_{ij} - \frac{C}{A}w_{ij} - CTI \leq 0
\]

\[\forall ij \in \mathcal{R}, \forall k \in \mathcal{B}_j, f=1,\ldots,N_F.
\]

Furthermore, the bounds of permissible pickup current setting values are expressed as follows, making use of (9) and (17):

\[w_{ij} - (z_{ij,f}^{\max} - 1)y_{ij,f} \leq 0\]

\[-w_{ij} + (z_{ij,f}^{\min} - 1)y_{ij,f} \leq 0\]

\[\forall ij \in \mathcal{R}, f=1,\ldots,N_F
\]

where, \(z_{ij,f}^{\min} = \left(I_{f,ij} / I_{pu,ij}^{\min}\right)^a\) and \(z_{ij,f}^{\max} = \left(I_{f,ij} / I_{pu,ij}^{\max}\right)^a\).

4) Quadratic Equality Constraints: Since pickup current settings are no longer design variables, the problem formulation heretofore does not assure that the obtained pickup current setting of each DOCR is the same, when the relay operates as primary or as backup. This constraint is written explicitly as:

\[I_{pu,ij} = I_{pu,ij}, \forall ij \in \mathcal{R}, \forall l \in \mathcal{C}_{ij}.
\]

Combining (17), (21) and (26), the following quadratic equation constraints are derived:

\[w_{ij}y_{ij,ij}^f - r_{ij,f}w_{ij}y_{ij,ij}^f + (1 - r_{ij,f})y_{ij,f}y_{ij,ij}^f = 0
\]

\[\forall ij \in \mathcal{R}, \forall l \in \mathcal{C}_{ij}, f=1,\ldots,N_F
\]

where \(r_{ij,f} = \left(I_{f,ij} / I_{pu,ij}^{\min}\right)^a\) are constant quantities. The constraints above compose a set of quadratic expressions, of the form \(x^T Q x = 0\), where \(x\) is the \(n \times 1\) vector of design variables, and \(Q\) is a real \(n \times n\) symmetric matrix.

As can be observed, optimized quantities for the suggested QCQP coordination problem are the variables \(w_{ij}, y_{ij,f}, y_{ij,ij}^f\), which represent the TD setting and operating time of DOCRs, respectively. Then, the obtained optimal values are used in (17) to derive the pickup current setting of each DOCR.
III. SOLVING THE QCQP OPTIMIZATION PROBLEM

The class of QCQP problems is a subset of NLP problems that goes beyond conventional quadratic programming (QP), having the following form [37], [38]:

$$\begin{align*}
\min_{x} & \quad x^{T}Q_{0}x + a_{0}^{T}x \\
\text{s.t.} & \quad x^{T}Q_{i}x + a_{i}^{T}x \leq b_{i}, \quad i \in I \\
& \quad x^{T}Q_{i}x + a_{i}^{T}x = b_{i}, \quad i \in E \\
& \quad l \leq x \leq u,
\end{align*}$$

(28)

where $x$, $a_{0}$, and $a_{i}$ are $n \times 1$ real vectors, and $Q_{0}$, $Q_{i}$ are $n \times n$ real symmetric matrices. In general, QCQP problems are smooth, non-convex, NP-hard problems [37]. In the particular case where $Q_{0}$ and $Q_{i}$, $i \in I$ are positive semidefinite and $Q_{i} = 0$, $i \in E$, the QCQP problem in (28) becomes convex, and can be efficiently solved to the global optimum [38]. Nevertheless, in the proposed formulation, $Q_{0} = 0$, $Q_{i} = 0$, $i \in I$, $a_{0} = 0$, $i \in E$, and $b_{i} = 0$, $i \in I \cup E$, which imply that the QCQP coordination problem is non-convex.

There are many commercial and free for academic purposes optimization tools, which can solve merely convex QCQP problems to global optimality, being insufficient to handle the general case of non-convex QCQP problems. Alternatively, NLP optimization solvers can be used to find local solutions. However, the behavior of the NLP algorithms depends on the appropriate choice of starting search point, and thus, it is quite difficult to assess obtained solutions.

In this paper, the proposed QCQP model is solved by the scip optimization suite [39], which is one of the fastest non-commercial solvers for mixed integer linear programs (MILPs). In particular, scip solves efficiently both convex and non-convex QCQP problems to global optimality in a deterministic manner, without dependence on the initial point [40]. In general, optimization software can handle problems with linear and quadratic constraints in a more effective manner than an NLP problem of the same scale. Indeed, evaluation of gradients and hessian matrices becomes simpler for the proposed model, facilitating the convergence of the algorithm to superior minimizers.

The new QCQP model has been implemented in MATLAB environment where the scip solver is called via the opensource toolbox OPTI [41], which provides a large number of optimization solvers as pre-compiled mex files. Furthermore, OPTI has the exceptional feature of generating and automatically supplying the exact first derivatives, which can lead to significant improvement in terms of convergence, accuracy, and computational cost.

IV. SIMULATION RESULTS AND DISCUSSION

The proposed QCQP formulation of the DOCR coordination problem has induced a great enhancement to the efficacy of the utilized optimization software. This assertion was verified by a series of simulations, executed on a dual-core 1.73-GHz PC with 2-GB RAM. For the sake of clear comparison, the DOCR coordination problem was firstly solved by scip solver using the new QCQP model, and then by knitro and fmincon solvers [42], [43], using the classical NLP model. Knitro and fmincon are powerful and versatile solvers for large-scale optimization problems, implementing algorithms which can guarantee local optimum solutions. Obtained simulation results demonstrate the expected superiority of the QCQP method over the prevailing NLP technique. It should be noted that the same initial search point $x_{0}$ was assumed for both cases.

The QCQP coordination problem was solved for several test systems with various configurations, considering symmetrical three-phase short circuits at different fault locations. In this section, the results of three coordination studies are presented and discussed. The coordination studies deal with the phase overcurrent elements of DOCRs, which use the IEC C1 Standard Inverse characteristic, defined by (8) with $A=0.14$, $B=0.02$, and $C=0$. It should be mentioned that the same procedure is followed for the coordination of the residual (or ground) elements, considering unsymmetrical ground faults.

Simulated DOCRs have 5A secondary nominal current and the $CTI$ is assumed to be equal to 0.3s. Moreover, the acceptable ranges concerning time dial setting and operating time of each relay, whether acting as primary or as backup, are determined by the following double inequalities, according to the considered values in relative literature:

$$\begin{align*}
0.1 \leq TD_{ij} & \leq 1.1 \Rightarrow 0.014 \leq w_{ij} \leq 0.154, \forall ij \in \mathcal{R}, \\
0.1s & \leq t_{ij}, t_{ij}^{\prime} \leq 1.1s \Rightarrow 0.1 \leq y_{ij}, y_{ij}^{\prime} \leq 1.1, \forall ij \in \mathcal{R}, k \in \mathcal{B}_{ij}.
\end{align*}$$

(29)

A. Illustrative Example: 3-bus Test System

The proposed method is illustrated using the 3-bus system, shown in Fig. 2. The data of this test system are given in [19] while the midline short-circuit values can be found in [30].

The minimum pickup current settings were taken equal to 1.5A, and its nominal secondary current was considered as the maximum acceptable value.

1) Design variables: The vector $x$ of design variables is given as follows:

$$
\begin{bmatrix}
w_{12} \\
w_{21} \\
w_{23} \\
w_{32} \\
w_{31} \\
y_{12} \\
y_{21} \\
y_{23} \\
y_{32} \\
y_{31} \\
y_{13}
\end{bmatrix}
$$

(30)

2) Objective function: The objective function is derived from (20) for $C=0$:

$$\begin{align*}
\min f = c^{T}x & = \sum_{y_{ij} \in \mathcal{R}} \left( y_{ij} + \sum_{k \in \mathcal{B}_{ij}} y_{ij}^{\prime} \right) = \\
& = y_{12} + y_{21}^{12} + y_{23}^{21} + y_{32}^{23} + y_{13}^{12} \\
& + y_{32}^{23} + y_{31}^{32} + y_{31}^{31} + y_{13} + y_{13}^{13}
\end{align*}$$

(31)

where $c^{T} = [0 0 0 0 0 1 1 1 1 1 1 1 1 1]$. 

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In (27), the following coordination criteria, is obtained from (24) as follows:

\[
\begin{align*}
\forall i,j \in R \Rightarrow \quad & y_{ij} - y_{ji} + \text{CTI} \leq 0 \\
& y_{ij} - \sum_{k=1}^{G_i} y_{ik} + 0.3 \leq 0
\end{align*}
\]

whereas the second set, with respect to the bounds of pickup current settings, is derived from (25):

\[
\begin{align*}
& w_{i1} - 0.0638y_{12} \leq 0 \\
& w_{21} - 0.0668y_{21} \leq 0 \\
& w_{32} - 0.0690y_{23} \leq 0 \\
& w_{32} - 0.0619y_{32} \leq 0' \\
& w_{13} - 0.0665y_{31} \leq 0 \\
& w_{13} - 0.0553y_{13} \leq 0
\end{align*}
\]

4) Quadratic equality constraints: Substituting the known values of parameters \( r_{ij} \), \( \forall i,j \in R \) in (27), the following quadratic constraints are derived:

\[
\begin{align*}
& w_{12}y_{12}^{13} - 1.0236w_{12}y_{12} - 0.0236y_{12}y_{12}^{23} = 0 \\
& w_{21}y_{21}^{13} - 1.0481w_{21}y_{21} - 0.0481y_{21}y_{21}^{13} = 0 \\
& w_{23}y_{23}^{13} - 1.0300w_{23}y_{23} - 0.0300y_{23}y_{23}^{23} = 0 \\
& w_{32}y_{32}^{13} - 1.0244w_{32}y_{32} - 0.0244y_{32}y_{32}^{23} = 0 \\
& w_{31}y_{31}^{12} - 1.0439w_{31}y_{31} - 0.0439y_{31}y_{31}^{23} = 0 \\
& w_{13}y_{13}^{23} - 1.0270w_{13}y_{13} - 0.0270y_{13}y_{13}^{23} = 0
\end{align*}
\]

5) Results: The simulation results, regarding the DOCR optimal setting values, are gathered in Table I. Moreover, Table II contains the operating times of all possible primary and backup relay pairs, and also the value of the objective function for both studied scenarios. As can be clearly observed, scip solved the problem to global optimality, while knitro and fmincon were trapped into the same local optimum. This fact proves the enhanced efficiency of the new method, owing to the QCQP problem formulation. Regarding the convergence time, scip spent only 0.08s to reach the global optimum.

### B. Case 2: 8-Bus Test System

This test case studies the coordination of DOCRs installed in the 8-bus system (Fig. 3). The system data are given in [23], while close-in short-circuit values were taken from [30]. Respecting the permissible limits of pickup current settings, the same boundary values as in the previous test case were considered.

In Table III, the obtained simulation results are presented. Similarly to the first case, it is obvious that the proposed QCQP approach has led to superior results, as expected, since scip guarantees globally optimal solution for the QCQP problem. Moreover, the convergence to the global optimum was attained after 39s, which is a very reasonable execution time, realizing the complexity of finding globally optimal solutions.

### C. Case 3: IEEE 30-Bus Network with Distributed Generation (DG)

This test case regards a large-scale optimization problem, where 38 DOCRs are located in the DG penetrated distribution portion of the IEEE 30-bus system. The short-circuit current values for close-in faults can be found in [11]. All buses of the network in Fig. 4 are numbered according to [44]. The ratio of current transformers was assumed equal to 1000/5. The upper and lower bounds for pickup current settings were taken equal to 1.5 A and 6 A respectively.

The QCQP problem comprises 134 design variables, 134 linear inequality constraints, and 58 quadratic equality constraints. Numerical simulation results for both QCQP and NLP models are given in Table IV. Obviously, scip solver succeeded in finding better minimizer than knitro and fmincon, for the same optimization problem, verifying the expected improvement in aggregate DOCRs operating time.

---

TABLE I

<table>
<thead>
<tr>
<th>Relay ID (i,j)</th>
<th>scip (QCQP) I (<em>{pu,y}(A)) TD(</em>{y})</th>
<th>knitro, fmincon (NLP) I (<em>{pu,y}(A)) TD(</em>{y})</th>
</tr>
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<tr>
<td>(1,2)</td>
<td>2.862, 0.100, 1.500, 0.158</td>
<td></td>
</tr>
<tr>
<td>(1,3)</td>
<td>1.720, 0.100, 1.500, 0.100</td>
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<tr>
<td>(2,1)</td>
<td>1.500, 0.100, 1.500, 0.100</td>
<td></td>
</tr>
<tr>
<td>(2,3)</td>
<td>2.482, 0.100, 1.616, 0.138</td>
<td></td>
</tr>
<tr>
<td>(3,1)</td>
<td>1.500, 0.100, 1.500, 0.100</td>
<td></td>
</tr>
<tr>
<td>(3,2)</td>
<td>2.342, 0.100, 1.946, 0.133</td>
<td></td>
</tr>
</tbody>
</table>

TABLE II

<table>
<thead>
<tr>
<th>Relay Pairs</th>
<th>scip (QCQP) (<em>{ij}(s)) TD(</em>{ij})</th>
<th>knitro, fmincon (NLP) (<em>{ij}(s)) TD(</em>{ij})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,2) (3,1)</td>
<td>0.279, 0.647, 0.347, 0.647</td>
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</tr>
<tr>
<td>(1,3) (2,1)</td>
<td>0.267, 0.784, 0.253, 0.784</td>
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</tr>
<tr>
<td>(2,3) (1,2)</td>
<td>0.209, 0.509, 0.209, 0.597</td>
<td></td>
</tr>
<tr>
<td>(3,1) (2,3)</td>
<td>0.211, 0.511, 0.211, 0.532</td>
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</tr>
<tr>
<td>(3,2) (1,3)</td>
<td>0.267, 0.567, 0.331, 0.509</td>
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</tbody>
</table>

Objective function value (fval)

5.032 s 5.271 s
Fig. 3. Single-line diagram of the 8-bus system.

TABLE III
OPTIMAL DOCR SETTING VALUES FOR THE 8-BUS SYSTEM

<table>
<thead>
<tr>
<th>Relay ID (i,j)</th>
<th>scipy (QCQP)</th>
<th>knitro, fmincon (NLP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I_{pu,i,j}(A)$</td>
<td>$TD_g$</td>
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<tr>
<td>(2,3)</td>
<td>5.000</td>
<td>0.131</td>
</tr>
<tr>
<td>(2,6)</td>
<td>5.000</td>
<td>0.107</td>
</tr>
<tr>
<td>(2,8)</td>
<td>4.666</td>
<td>0.100</td>
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<td>(4,3)</td>
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<td>(4,5)</td>
<td>3.765</td>
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<tr>
<td>(5,4)</td>
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<td>(5,6)</td>
<td>1.861</td>
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<tr>
<td>(6,2)</td>
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<td>0.107</td>
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<td>0.132</td>
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<td>(8,6)</td>
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<tr>
<td>Objective function value (fval)</td>
<td>21.892 s</td>
<td>25.136 s</td>
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V. CONCLUSION

In this paper, a new optimization method was proposed to establish optimal coordination among directional overcurrent relays in power systems. The relay coordination problem was formulated as a quadratically constrained quadratic programming model, employing graph theory principles, and solved to global optimality utilizing the scipy suite. Simulations conducted on three different test systems demonstrate the superiority, as well as the enhanced performance, of the suggested model over the well-studied NLP approach.

REFERENCES


Fig. 4. The DG-penetrated 33-kV portion of the IEEE 30-bus system.

TABLE IV
OPTIMAL DOCR SETTING VALUES FOR THE 30-BUS SYSTEM

<table>
<thead>
<tr>
<th>Relay ID (i,j)</th>
<th>scipy (QCQP)</th>
<th>knitro, fmincon (NLP)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$I_{pu,i,j}(A)$</td>
<td>$TD_g$</td>
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<td>(10,17)</td>
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<tr>
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<td>(30,29)</td>
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<tr>
<td>Objective function value (fval)</td>
<td>64.710 s</td>
<td>67.366 s</td>
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</table>


