Optimizing Combined Emission Economic Dispatch for Solar Integrated Power Systems

Naveed Ahmed Khan, Guftaar Ahmad Sardar Sidhu, and Feifei Gao

Abstract—The dispatch of power at minimum operational cost of thermal energy sources has been a significant part of research since decades. Recently with increasing interests in renewable energy resources, the optimal economic dispatch has become a challenging issue. This paper presents combined emission economic dispatch (CEED) model for a solar photo voltaic (PV) integrated power system with multiple solar and thermal generating plants. We formulate mixed integer binary programming problem (MIBP) subject to various practical constraints. A decomposition framework is proposed where the original problem is split into two sub-problems. Particle swarm optimization (PSO), Newton-Raphson method, and binary integer programming (BIP) techniques are exploited to find the joint optimization solution. The proposed model is tested on IEEE 30 bus system. The simulation results demonstrate the effectiveness of the proposed model.

Index Terms—Economic Dispatch, Power Systems, Photo Voltaic, PSO

I. INTRODUCTION

In recent decades, deployment of renewable energy sources in place of thermal generation has received a lot of attention. This shift of momentum took place due to the rapidly depleting reserves of fossil fuels, the dramatic increase in fuel prices, and the environmental concerns associated with thermal fuels [1]. Economic dispatch (ED) is a well known optimization problem that aims to find an appropriate combination of power shares from committed generating units that results in minimum cost. There are numerous constraints involved in ED problem such as power limits of generators, power balance, prohibited operating zones, ramp rate limits etc. Various optimization techniques have been reported in literature to solve the ED problem [2].

ED with thermal generating units has been one of the most frequently investigated problems in power system optimization [3]–[4]. The structure of fuel cost functions for thermal generating units plays a significant role in ED problem [5]. Traditionally, the problem had been extensively studied with smooth convex cost functions and efficient solutions were proposed [6]. However, the complex nature of the commonly used generation facilities has made the cost functions essentially non-convex [7]. For instance, thermal units equipped with multi-valve steam turbines produce ripples in the cost functions. These ripples like effects are originated due to opening or closing of valves at various stages, and are known as the valve point effects (VPEs). In addition to VPEs, the aspect of multiple fuel options also results in non-smooth objective functions for ED optimization [8]. In literature, various artificial intelligence (AI) based optimization techniques were proposed to address the complex ED problem e.g., genetic algorithm (GA) [9], particle swarm optimization (PSO) [10], neural network (NN) [11], evolutionary programming (EP) [12] and tabu search (TS) [13].

PSO has been the most popular technique to address ED optimization due to its simplicity and ability to solve the complex problems [14]. In [15], the authors addressed non-convex problem by a hybrid scheme which combines PSO and sequential quadratic programming (SQP). The constraints like power balance, security, power generation limits, ramp rate limits, and spinning reserve were involved. PSO was used as a main optimization tool and SQP as a local optimizer for further tuning of the obtained solution. In [16], constriction factor PSO (CFPSO) had been proposed for non-convex ED under various constraints such as generator limits and power balance. The inclusion of constriction factor ensured fast convergence of the algorithm by gradually reducing the velocities. Considering more complex cost function under the prohibited operating zones and VPEs, a technique named ‘iteration PSO with time varying acceleration coefficients (IPSO-TVAC)’ was proposed in [17]. The iteration term in the velocity equation enhanced the memorizing ability of PSO whereas the time varying acceleration coefficients resulted in better balance between social and local components.

Thermal generating units produce emissions that result in serious environmental impacts [18]. Therefore, along with the cost minimization, a significant intention has been paid to keep the emissions at minimum level. A multi-objective ED problem, which involves both the fuel cost and emissions is known as combined emission economic dispatch (CEED). Selvakumar et al. [19] solved the CEED problem under the constraints of power balance and power generation capacity limits using PSO. The authors of work in [20] proposed multi-objective particle swarm optimization (MOPSO) and used a diversity-preserving mechanism to find the wide range

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of pareto-optimal solutions. Moreover, [20] considered the practical security constraints which were missing in [19]. In [21], the authors enhanced MOPSO procedure by redefining the local and global best individuals resulting in significant improvements. In [22], multi-objective CEED problem was converted into a single objective optimization by introducing a price penalty factor. The problem was solved using the artificial bee colony (ABC) algorithm. Further, a modified harmony search algorithm to solve the combined economic and emission load dispatch (CEELD) problem was designed in [23].

The ED of renewable integrated power system becomes more complex due to intermittent and uncontrollable nature of the renewable energy. The work in [24] proposed a two-step framework to evaluate the energy storage in a wind aided power system. The authors dealt with the wind intermittency using reserve generation. However, this work was limited to battery storage scenario and the actual network constraints were missing. In [25], the authors studied short term planning of virtual power plants (VPP) under the objectives of maximizing economic profit and minimizing deviations between actual and contracted energy. The scope of this paper was little to propose bid based operation of VPPs connected to a very small network. The authors in [26] presented a stochastic model to solve an optimal power flow (OPF) problem for wind integrated power systems. In this paper, the level of wind penetration was determined based on the demand-side flexibility. The work [24]–[26] did not consider the emissions and important constraints like voltage limits, tap limits, reactive power limits etc., were also ignored. In [27], the authors provided economic environmental dispatch (EED) model by integrating the renewable energy sources in conventional grid. The test system involved six thermal machines, one photo voltaic (PV) plant, and a single wind turbine. The problem was formulated subject to power balance, generator limits, and renewable energy limit constraints. A strength pareto evolutionary algorithm (SPEA) was used as an optimization tool. This work was extended in [28] and a dynamic economic emission dispatch (DEED) model was proposed considering the additional constraints of security, ramp rate limits, and line capacity. However, the considered test system was very small, with a limited penetration of renewable energy share and had only six buses. Moreover, the constraints such as reactive power limits, bus voltage limits, and tap changer limits were neglected. The work [29] proposed a hybrid model for off-grid applications to minimize the fuel and the battery wear costs subject to various constraints like availability of the PV power, the battery bank charge, and the load power demand. In [30], the authors investigated ED with high penetration of renewable energy resources and evaluated the effects of uncertainties and network congestion on the dispatch strategies. Further, [31] considered the solar integrated power system and proposed an efficient PSO based solution to a more general CEED optimization problem subject to the constraints of power balance, generator limits, renewable energy limits, and the ramp rate limits. However, the practical security and the network losses constraints were missing and the authors did not test the proposed solution on any real network. More recently, the work in [32] developed a rule-based power management algorithm to achieve dispatch characteristics similar to conventional thermal generating units. However, the study was limited to only a single PV plant without implementation on real network. Thus, various constrains of practical power system were absent.

In this paper, we present a joint CEED optimization problem which aims to 1) minimize the thermal fuel cost, the emissions, and the cost of solar generation, 2) maximize the share of the solar power and the number of participating solar units. We consider various practical network and security constraints i.e. power balance, generator bounds, bus voltage bounds, reactive power limits, line thermal capacity limits, available solar power, and upper bound on solar penetration. It should be noted that the existing works in literature may only have some of these constraints. The contributions of this paper are summarized as:

- A joint multi-objective integer optimization problem is formulated to minimize the overall operational cost/emissions and to maximize the total share/number of solar power plants.
- A decomposition framework is first adopted to split the original problem into two sub-problems. Then, a mixed optimization scheme is proposed which exploits the PSO, the Newton-Raphson method, and the binary integer programming.
- The proposed model is employed to IEEE 30-bus network to verify its practical application.
- Through simulations, we show the better performance of the proposed algorithm in terms of cost minimization and maximization of the solar share.

The rest of the paper is organized as follows: system model and joint optimization problem are presented in section II. Section III elaborates the proposed solution. Simulation results and discussions are given in section IV, and finally the conclusion is presented in section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a power generation supply system consisting of $n$ number of thermal and $m$ number of solar generating plants, supplying $NB$ number of loads as shown in Fig. 1. Let $P_l^i$ be the power generated by the $i^{th}$ thermal generating unit, the fuel cost in $$/h$ is expressed as [8]:

$$F_l(P_l) = a_lP_l^2 + b_lP_l + c_l + |c_i \sin(f_i \times (P_l^{min} - P_l))|$$

(1)

1. Maximizing the number of participating solar plants minimizes the probability of failure due to intermittent nature of solar radiation.

2. We assume that a feasible solution always exists in terms of demand and supply, that is to say the generating plants are assumed to satisfy the demands of users in the entire system.
where $a_i$, $b_i$, $c_i$, $e_i$, and $f_i$ are fuel cost coefficients of $i^{th}$ generating unit. $P_i^{min}$ is the minimum power limit of $i^{th}$ generating unit. Thermal generating units produce emissions which can be presented in kg/h by:

$$E_i(P_i) = \alpha_i P_i^2 + \beta_i P_i + \gamma_i + \varepsilon_i \exp(\delta_i \times P_i),$$  \hspace{1cm} (2)

where $\alpha_i$, $\beta_i$, $\gamma_i$, $\varepsilon_i$, and $\delta_i$ are emission coefficients of the $i^{th}$ generating unit. The sine and exponential terms in (1) and (2) represent VPEs.

Let $P_{gs_j}$ be the power generated by the $j^{th}$ solar plant, the cost of operation in $$/h$ is given by following expression,

$$G_j(P_{gs_j}) = PUCost_j \times P_{gs_j},$$  \hspace{1cm} (3)

where $PUCost_j$ is the per unit cost in $$/MWh of $j^{th}$ solar plant. $P_{gs_j}$ is given by [27]:

$$P_{gs_j} = Prated_j \left[1 + (T_{amb} - T_{ref}) \times \alpha_{p_{s_j}}\right] \times \frac{S_i}{1000},$$  \hspace{1cm} (4)

with $Prated_j$ being the rated power of $j^{th}$ solar plant, $T_{amb}$ the ambient temperature, $T_{ref}$ the reference temperature, $\alpha_{p_{s_j}}$ the temperature coefficient, and $S_i$ the incident solar radiation.

Each thermal generating unit has its own range of real power generation $P_i$ and reactive power generation $Q_i$, i.e.,

$$P_i^{min} \leq P_i \leq P_i^{max} \quad \forall i = 1, 2, ..., n,$$  \hspace{1cm} (5)

$$Q_i^{min} \leq Q_i \leq Q_i^{max} \quad \forall i = 1, 2, ..., n,$$  \hspace{1cm} (6)

where $P_i^{max}$, $Q_i^{min}$, and $Q_i^{max}$ are the maximum real power limit, minimum reactive power limit, and maximum reactive power limit of the $i^{th}$ generating unit, respectively.

All the generating plants and loads are assumed to be connected to IEEE 30-bus network, which has its own limitations.

At any time during operation, the voltage magnitude $V_k$ at any bus $k$ should remain within specified limits,

$$V_k^{min} \leq V_k \leq V_k^{max} \quad \forall k = 1, 2, ..., NB,$$  \hspace{1cm} (7)

where $NB$ are the total number of buses. The unknown parameters of network are found from standard power flow equations:

$$\overline{P}_k = \sum_{s=1}^{NB} [V_k || V_{k,s} || |Y_{k,s}| \cos(\theta_{k,s} - \zeta_k + \zeta_s)],$$  \hspace{1cm} (8)

$$\overline{Q}_k = \sum_{s=1}^{NB} [V_k || V_{k,s} || |Y_{k,s}| \sin(\theta_{k,s} - \zeta_k + \zeta_s)],$$  \hspace{1cm} (9)

where $\overline{P}_k$ and $\overline{Q}_k$ are the real and reactive power injections at bus $k$, respectively, $Y_{k,s}$ is the $(k, s)^{th}$ element of admittance matrix $Y$, $\theta_{k,s}$ is the angle of the admittance element $Y_{k,s}$, $\zeta_k$ is the angle of respective voltage, and $|| \cdot ||$ represents the magnitude. Tap changing transformers for voltage adjustments are equipped with limited number of taps. Therefore the selected tap $T_t$ of $t^{th}$ transformer should be within the range of available taps:

$$T_t^{min} \leq T_t \leq T_t^{max} \quad \forall t = 1, 2, ..., NT,$$  \hspace{1cm} (10)

with $NT$ be the total number of transformers. Also the power flow $S_{lz}(k, s)$ at any line $z$ connected between buses $k$ and $s$ should not exceed its capacity, i.e.,

$$S_{lz}(k, s) \leq S_{lz}^{max}(k, s) \quad \forall z = 1, 2, ..., NZ,$$  \hspace{1cm} (11)

where $NZ$ is total number of lines. $S_{lz}(k, s)$ can be obtained as:

$$S_{lz}(k, s) = V_k \times (V_k - V_s)^* \times (Y_{k,s})^* + Y_{k,0} \times V_k^2,$$  \hspace{1cm} (12)

where $(x)^*$ represents the complex conjugate of $x$.

B. Problem Formulation

This work deals with the CEED problem incorporating both the thermal and solar power generations. It aims to reduce the overall costs and the thermal emissions. Another target is to get the maximum benefits from the installed solar capacity by maximizing the solar share and to decrease the probability of solar failure by increasing the number of participating solar units. The multi-objective optimization can be formulated.
mathematically, as:

$$
\begin{align*}
\min_{P_i, U_{sj}, P_{sk}} & \sum_{i=1}^{n} (F_i(P_i) + E_i(P_i)) + \sum_{j=1}^{m} G_j U_{sj} \\
& - \sum_{k=1}^{NB} P_{sk} - \sum_{j=1}^{NB} U_{sj} \\
\text{s.t.,} & \sum_{k=1}^{NB} P_{dk} + \sum_{z=1}^{NL} P_{l_z} - \sum_{i=1}^{n} P_i - \sum_{k=1}^{NB} P_{sk} = 0, \\
& \sum_{j=1}^{m} P_{gsj} U_{sj} \leq \sum_{k=1}^{NB} P_{dk}, \\
& (5), (6), (7), (10), \text{ and } (11).
\end{align*}
$$

where $U_{sj}$ is a binary variable that represents ON or OFF state of the $j$th solar plant. $P_{dk}$ represents the power demand at the $k$th bus. $P_{l_z}$ is power loss over the $z$th line, and $\Gamma$ defines the maximum limit of the solar power based on available reserve capacity. The constraint in equation (14) is known as power balance and states that the total generated power must cope with the total demands plus network losses; whereas according to constraint (15), the share of the solar power should be less than or equal to the maximum limit.

### III. PROPOSED SOLUTION

The optimization in (13) is a mixed integer binary programming problem (MIBP). To facilitate the solution, we define a variable $P_{sk}$ that denotes the total share of solar power over the bus $k$. With this, the optimization problem (13) can be reformulated as

$$
\begin{align*}
\min_{P_i, U_{sj}, P_{sk}} & \sum_{i=1}^{n} (F_i(P_i) + E_i(P_i)) + \sum_{j=1}^{m} G_j U_{sj} \\
& - \sum_{k=1}^{NB} P_{sk} - \sum_{j=1}^{NB} U_{sj} \\
\text{s.t.,} & \sum_{k=1}^{NB} P_{dk} + \sum_{z=1}^{NL} P_{l_z} - \sum_{i=1}^{n} P_i - \sum_{k=1}^{NB} P_{sk} = 0, \\
& 0 \leq \sum_{k=1}^{NB} P_{sk} \leq \sum_{j=1}^{m} P_{gsj} \Gamma \sum_{k=1}^{NB} P_{dk}, \\
& \sum_{j=1}^{m} P_{gsj} U_{sj} = \sum_{k=1}^{NB} P_{sk}, \forall U_{sj} \in \{0,1\}, \\
& (5), (6), (7), (10), \text{ and } (11).
\end{align*}
$$

The structure of the problem in (17) allows to decompose it into two independent sub-problems without loss of optimality. The two sub-problems and their corresponding solutions are given in the following subsections.

### A. Sub-Problem I

The first sub-problem aims to minimize the cost of thermal share and maximize the solar share subject to corresponding constraints. Thus the optimization becomes

$$
\begin{align*}
F_1 = \min_{P_i, P_{sk}} & \sum_{i=1}^{n} (F_i(P_i) + E_i(P_i)) - \sum_{k=1}^{NB} P_{sk} \\
\text{s.t.,} & (5), (6), (7), (10), (11), (18), \text{ and } (19).
\end{align*}
$$

To transform $E_i(P_i)$ into emission cost, we introduce a price penalty factor $h_i$ such that

$$
h_i = \frac{a_i P_{i max}^2 + b_i P_{i max} + c_i + \epsilon_i \times \sin(f_i \times (P_{i min} - P_{i max}))}{\alpha_i P_{i max} + \beta_i P_{i max} + \gamma_i + \epsilon_i \times \exp(\delta_i \times P_{i max})}.
$$

Further, for tractability of solution we replace $P_{sk}$ with $P_{sk}^2$, since for $P_{sk} \geq 0$ minimizing $P_{sk}$ is equivalent to minimizing $P_{sk}^2$. Thus we have

$$
\begin{align*}
F_1 = \min_{P_i, P_{sk}} & \sum_{i=1}^{n} (F_i(P_i) + h_i E_i(P_i)) - \sum_{k=1}^{NB} P_{sk}^2 \\
\text{s.t.,} & (5), (6), (7), (10), (11), (18), \text{ and } (19).
\end{align*}
$$

To solve this problem, we propose a mixed optimization scheme based on the PSO and the Newton Raphson method. The steps in the proposed algorithm are:

1. Initialize swarm size, $\omega_{min}$, $\omega_{max}$, $\omega^y = \omega_{max}$, $iter_{max}$, $v_{x min}$, and $v_{x max}$; where $y = 1$ is the current iteration number, $\omega_{min}$ and $\omega_{max}$ are the minimum and the maximum limits of inertia weight, respectively, $\omega^y$ is the inertia weight at current iteration, and $v_{x min}$ and $v_{x max}$ are the minimum and maximum limits of the velocity. Randomly generate $P_i$, $P_{sk}$, and $v_i$ such that:

$$
P_i^{min} \leq P_i \leq P_i^{max} \quad \forall i = 1, 2, ..., n, \\
0 \leq \sum_{k=1}^{NB} P_{sk} \leq \min \left( \sum_{j=1}^{m} P_{gsj} \Gamma \sum_{k=1}^{NB} P_{dk} \right), \\
v_i^{min} \leq v_i \leq v_i^{max} \quad \forall i = 1, 2, ..., n.
$$

2. For a problem with $n$ thermal units and $S$ solar shares, generate the position $P_{swarm}$ and velocity $v_{swarm}$ of each particle in swarm such that:

$$
P_{swarm} = [P_{ts1} P_{ts2} ... P_{tsx} ... P_{tsSS}]^T, \\
v_{swarm} = [v_{t1} v_{t2} ... v_{tx} ... v_{tSS}]^T,
$$

where $P_{tsx}$ and $v_{tsx}$ are the position and velocity vectors of $x$th particle and are given by:

$$
P_{tsx} = [P_1 P_2 ... P_n P_{ts2} ... P_{tsSS}], \\
v_{tsx} = [v_{t1} v_{t2} ... v_{tn} v_{ts1} v_{ts2} ... v_{tSS}].
$$

$P_{tsx}$ is a vector of decision variables in sub-problem I.

3. Obtain the power flows (bus voltage magnitudes, line power flows, and line losses) for each generated $P_{tsx}$.
using Newton Raphson method such that the constraints in (6), (7), and (10) are satisfied. Evaluate the fitness function as

\[ F^y_{\text{swarm}} = [\hat{F}(P_{\text{ts}1}) \hat{F}(P_{\text{ts}2}) ... \hat{F}(P_{\text{ts}x}) ... \hat{F}(P_{\text{ts}SS})]' , \]

where \( \hat{F} \) is given by

\[ \hat{F} = \sum_{i=1}^{n} (F_i(\hat{P}_i) + h_iE_i(\hat{P}_i)) - \sum_{k} \hat{P}_s^2_k. \]  

(34)

Set \( P_{\text{best}} = P^y_{\text{swarm}}, g_{\text{best}} = P^y_{\text{swarm}}(\text{ind}_g), F_{g_{\text{best}}} = F^y_{\text{swarm}}, \) and \( F_{g_{\text{best}}} = \min_x F^y_{\text{swarm}} \), where \( P^y_{\text{swarm}}, P_{\text{best}}, g_{\text{best}}, F_{\text{best}}, \) and \( F_{g_{\text{best}}} \) are the fitness evaluation at current position, personal best position of individual particles, global best position, fitness evaluation at personal best position, and fitness evaluation at global best position, respectively. The \( \text{ind}_g \) is the index of minimum value in \( F^y_{\text{swarm}} \), i.e.,

\[ \text{ind}_g = (\arg \min_x (F^y_{\text{swarm}})). \]  

(35)

(4) Update the velocity of each particle as:

\[ v_{tsx}^{y+1} = \begin{cases} \max(xv_{y+1}, v_{x}^{\min}), & \text{if } xv_{y+1} < v_{x}^{\max}, \\ \min(xv_{y+1}, v_{x}^{\max}), & \text{if } xv_{y+1} > v_{x}^{\max}, \end{cases} \]

with

\[ xv_{y+1} = \omega^y v_{tsx}^y + C_1r_1(P_{\text{best}}^x - P_{tsx}^y) + \]

\[ C_2r_2(g_{\text{best}}^x - P_{tsx}^y), \]

(37)

where \( xv_{y+1} \) is velocity of \( x^{th} \) particle at iteration \( y+1 \), \( \omega^y \) is a parameter known as inertia weight at iteration \( y \), \( C_1 \) and \( C_2 \) are acceleration coefficients, whereas \( r_1 \) and \( r_2 \) are two random numbers between 0 and 1.

(5) Update the position of each particle as

\[ P_{tsx}^{y+1} = \begin{cases} \max(xx_{y+1}, P_{tsx}^{\min}), & \text{if } xx_{y+1} < P_{tsx}^{\max}, \\ \min(xx_{y+1}, P_{tsx}^{\max}), & \text{if } xx_{y+1} > P_{tsx}^{\max}, \end{cases} \]

where \( xx_{y+1} \) is given by,

\[ xx_{y+1} = P_{tsx}^y + v_{tsx}^{y+1}. \]  

(39)

(6) Find the power flows and evaluate the fitness \( F^y_{\text{swarm}} = F_1(P_{\text{ts}}) \) and update \( F_{g_{\text{best}}} = \min_x F^y_{\text{swarm}} \). Obtain \( g_{\text{best}} \) and \( P_{\text{best}} \) as

\[ g_{\text{best}} = P^y_{\text{swarm}}(\text{ind}_g), \]

\[ P_{\text{best}} = PTB^x(\text{ind}_g), \]

(40)

(41)

where

\[ P_{TB}^x = [P^x_{\text{swarm}} P^x_{\text{best}}], \quad \forall x, \]  

(42)

\[ \text{ind}_g = \arg \min_x F^y_{\text{swarm}}, \]  

(43)

\[ \text{ind}_p = \arg \min_x (F^x_{\text{swarm}}, F^x_{\text{best}}), \quad \forall x. \]  

(44)

(7) Update the inertia weight and increment the iteration number, accordingly, as

\[ \omega^{y+1} = \omega_{\text{max}} - (\omega_{\text{max}} - \omega_{\text{min}})/\text{iter}_{\text{max}} \times y, \]

\[ y = y + 1. \]

(8) Repeat steps (4) to (7) until convergence.

B. Sub-Problem II

The objective of second problem is to minimize the cost of the solar power along with the maximization of the number of solar plants. The problem can be written as:

\[ F_2 = \min_{U_s} \sum_{j=1}^{m} P_{\text{UCost}} j \times P_{gs} j \times U_s j - \sum_{j=1}^{m} U_s j \]  

s.t., \[ \sum_{j=1}^{m} P_{gs} j \times U_s j = \sum_{k} P_{sk}. \]

The problem can be rewritten as:

\[ F_2 = \min_{U_s} c_1 U_s^T - I_1 U_s^T, \]  

s.t., \[ c_2 U_s^T = I_1 c_3^T. \]

(45)

(46)

(47)

where \( c_1 \) is a vector of cost coefficients, \( I_1 \) is identity vector, and \( c_4 \) is a vector of solar powers. This is a standard integer programming problem and Binary Integer Programming toolbox in MATLAB is used to obtain the solution. The toolbox takes the coefficients of objective function as well as of constraints as input and provides the binary decision variables and the corresponding fitness evaluation as output.

This completes the solution of optimization in (13). For more clarity, the detailed steps of proposed joint optimization solution are summarized in algorithm 1.

**Algorithm 1 Proposed Joint Optimization Solution**

1. Initialize swarm size, \( \omega_{\text{min}}, \omega_{\text{max}}, \omega^y = \omega_{\text{max}}, \text{iter}_{\text{max}}, v_{x}^{\min}, v_{x}^{\max} \). Generate \( \hat{P}_i, P_{sk}, \) and \( v_i \) according to the equations (26), (27), and (28). Set \( y = 1 \).

2. Generate \( P^y_{\text{swarm}} \) and \( v^y_{\text{swarm}} \), such that the equations (5), (19), and (28) are satisfied.

3. Obtain the initial power flows from Newton Raphson method as discussed in III-A.

4. After step 8 of the solution proposed in III-A, set \( P^*_{i} = \hat{P}_i \) and \( P_{sk}^* = P_{sk} \).

5. For the obtained \( P_{sk}^* \), solve the sub-problem II in (46) using Binary Integer Programming toolbox of MATLAB.

6. Terminate the procedure and display the results.
IV. TEST SYSTEM AND SIMULATION RESULTS

The proposed model is implemented on standard IEEE 30 bus system with 6 thermal machines and 13 solar plants. The data for the thermal units and the power demand, for the IEEE 30 bus system, and for the solar plants are taken from [31], [33], and Table I, respectively. The bus number 1 in IEEE network is considered as the slack bus. The machine number 5, which is the largest machine, is connected to bus 1. The machines 1, 2, 3, 4, and 6 are connected to buses 2, 5, 6, 8, and 11, respectively. The solar units are collectively connected to bus 15. The value of $\Gamma$ is set to 0.3 [27] and the constants $C_1$ and $C_2$ in cognitive and social components of velocity equation are selected to be 2. Furthermore, maximum number of iterations and swarm size are taken to be 200 and 50 respectively. The results are presented for 6 hours of the day from 10:00 to 15:00 hours.

Table II presents the obtained thermal generation, solar share, cost, emissions, and network losses etc., for the hours under consideration. Thermal generation includes individual powers of thermal units that result in minimum value of the fitness function. Solar generation illustrates the maximum possible share that can be obtained subject to the considered constraints. It can be observed that the obtained power generations from all the thermal units are within their lower and upper bounds. The fuel cost varies with the share of thermal generation i.e., a large share results in higher cost and vice versa. Similarly, the same happens for the case of emissions. It is also evident from the table that the power mismatch values are significantly small. This in turn reveals the satisfactory performance of the proposed model in terms of fitness value versus iterations. It is evident that the algorithm is well converged within few number of iterations i.e., 120 iterations at the maximum. Further, Fig. 3 shows the capacity violation of lines at different hours. It is clear from the results that there exists considerably large remaining capacity (i.e., line capacity minus line power flows) on all the lines, having small violations at few lines. The violation at line 13 is due to the algorithmic dictation to have a relatively larger share from the thermal unit connected to bus 11 to get the minimum fitness evaluation. The objective of solar share maximization at bus 15 results in small violations at the lines 22 and 30.

<table>
<thead>
<tr>
<th>Plant #</th>
<th>$P_{\text{rated}}$ (MW)</th>
<th>Unit Rate ($$/KWh)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>0.22</td>
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<td>6</td>
<td>35</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
<td>0.26</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>0.27</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>0.27</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>0.275</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
<td>0.28</td>
</tr>
<tr>
<td>12</td>
<td>40</td>
<td>0.28</td>
</tr>
<tr>
<td>13</td>
<td>40</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table III shows the ON/OFF status of the solar power plants at different hours. It is worth noting that solar plant 1 is the largest and the least expensive plant (table I) among all the 13 solar plants. Hence it should be the first choice to minimize the overall cost. Surprisingly, on the other hand it is OFF in all the hours. This is because of the objective of maximizing the number of solar plants i.e., selecting the largest solar plant will result in decreasing the number of plants.

Figure 2 depicts the convergence properties of the proposed model. In this figure, the convergence is shown in terms of fitness value versus iterations. It is evident that the algorithm is well converged within few number of iterations i.e., 120 iterations at the maximum. Further, Fig. 3 shows the capacity violation of lines at different hours. It is clear from the results that there exists considerably large remaining capacity (i.e., line capacity minus line power flows) on all the lines, having small violations at few lines. The violation at line 13 is due to the algorithmic dictation to have a relatively larger share from the thermal unit connected to bus 11 to get the minimum fitness evaluation. The objective of solar share maximization at bus 15 results in small violations at the lines 22 and 30.

**TABLE III: Status of solar plants at hours under consideration.**

<table>
<thead>
<tr>
<th>Time of the day (h)</th>
<th>10:00</th>
<th>11:00</th>
<th>12:00</th>
<th>13:00</th>
<th>14:00</th>
<th>15:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{81}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$U_{82}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{83}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{84}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{85}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{86}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{87}$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$U_{88}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{89}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{90}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{91}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$U_{92}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$U_{93}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 4 presents results of bus voltages versus iterations for 12:00 hours. Despite the little variations, the voltage magnitudes are always within specified limits (0.95 pu to 1.10 pu) at each iteration. This is guaranteed by the nested Newton-Raphson method within each iteration of the PSO. The bus voltages exhibit similar behavior for other hours and are not given for simplicity.
TABLE II: Results computed for the time under consideration.

<table>
<thead>
<tr>
<th>Time of the day</th>
<th>10:00 hrs</th>
<th>11:00 hrs</th>
<th>12:00 hrs</th>
<th>13:00 hrs</th>
<th>14:00 hrs</th>
<th>15:00 hrs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong> <em>1</em> (10 MW-125 MW)</td>
<td>69.4351</td>
<td>49.8071</td>
<td>69.1978</td>
<td>56.0319</td>
<td>91.1074</td>
<td>48.7134</td>
</tr>
<tr>
<td><strong>P</strong> <em>2</em> (10 MW-150 MW)</td>
<td>105.9139</td>
<td>77.6055</td>
<td>104.7843</td>
<td>85.9219</td>
<td>124.4405</td>
<td>74.9303</td>
</tr>
<tr>
<td><strong>P</strong> <em>3</em> (40 MW-250 MW)</td>
<td>229.9159</td>
<td>166.0272</td>
<td>228.24</td>
<td>184.5236</td>
<td>214.4405</td>
<td>160.7347</td>
</tr>
<tr>
<td><strong>P</strong> <em>4</em> (35 MW-210 MW)</td>
<td>192.184</td>
<td>171.8485</td>
<td>191.5725</td>
<td>178.3799</td>
<td>210.6234</td>
<td>169.2724</td>
</tr>
<tr>
<td><strong>P</strong> <em>5</em> (130 MW-325 MW)</td>
<td>223.4905</td>
<td>185.4724</td>
<td>222.8431</td>
<td>197.2717</td>
<td>263.5748</td>
<td>184.0694</td>
</tr>
<tr>
<td><strong>P</strong> <em>6</em> (125 MW-315 MW)</td>
<td>195.6107</td>
<td>195.4755</td>
<td>195.5971</td>
<td>195.5166</td>
<td>195.8192</td>
<td>195.4356</td>
</tr>
<tr>
<td><strong>Thermal Generation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>P</strong> <em>3</em> (MW)</td>
<td>250.5754</td>
<td>262.7725</td>
<td>259.0267</td>
<td>244.0533</td>
<td>261.5668</td>
<td></td>
</tr>
<tr>
<td><strong>Cost</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel Cost ($/h)</td>
<td>5.28E+04</td>
<td>4.38E+04</td>
<td>5.26E+04</td>
<td>4.64E+04</td>
<td>5.75E+04</td>
<td>4.31E+04</td>
</tr>
<tr>
<td>Total Cost ($/h)</td>
<td>1.19E+05</td>
<td>1.12E+05</td>
<td>1.16E+05</td>
<td>1.12E+05</td>
<td>1.20E+05</td>
<td>1.11E+05</td>
</tr>
<tr>
<td><strong>Solar Generation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>P</strong> <em>s</em> (MW)</td>
<td>894.178</td>
<td>617.0894</td>
<td>886.0768</td>
<td>691.4097</td>
<td>1.01E+03</td>
<td>598.4498</td>
</tr>
<tr>
<td><strong>Power Mismatch (MW)</strong></td>
<td>1.14E-13</td>
<td>2.27E-13</td>
<td>1.95E-13</td>
<td>-7.46E-14</td>
<td>3.16E-13</td>
<td>3.55E-14</td>
</tr>
</tbody>
</table>

Fig. 3: Capacities and power flows of the lines.

Figure 5 shows the total solar cost, the solar share, and the number of selected plants in different hours. It can be noticed that the total solar cost may come out to be different for the same number of solar plants and is directly proportional to the solar share. For instance, the number of solar plants at 11:00 hours and 12:00 hours are same; however, the cost at the 12:00 hours is less as compared to 11:00 hours. This is because of the selection of different solar plants (i.e., 3, 4, 5, 6, 7, 8, and 9 instead of 4, 5, 6, 8, 9, 10, and 13) resulting in different total share. Lastly, we analyze the generation of emissions with the variations in solar and the thermal shares in Fig. 6. As expected, the emissions increase with the increase in thermal share and vice versa. Similarly, the emissions decrease with an increase in solar share, i.e., the amount of emission is inversely proportional to the total solar share.

V. CONCLUSION

This work presented a new CEED model for a solar PV integrated power system which could minimize the fuel cost, emissions, solar cost, and could maximize the solar share and the number of solar plants. The constraints of power balance and the bounds on generators, renewable energy, voltage magnitudes, transformer taps, and line capacities were considered in the joint optimization problem. The amount of solar share was subject to various network constraints, renewable energy limit, and available solar power. A hybrid optimization scheme was proposed using the PSO, the Newton method, and the binary integer programming. The results showed the promising performance of the proposed algorithm satisfying all constraints with an acceptable complexity in terms of convergence.

REFERENCES

Fig. 5: Effects of solar share and selected solar plants on solar cost.

Fig. 6: Level of emissions for various thermal and solar shares.


