Optimal power flow with emission and non-smooth cost functions using backtracking search optimization algorithm

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ABSTRACT

Economic operation of electric energy generating systems is one of the prevailing problems in energy systems. In this paper, a new method called the Backtracking Search Optimization Algorithm (BSA) is proposed for solving the optimal power flow problem. This method is tested for 16 different cases on the IEEE 30-bus, IEEE 57-bus, and IEEE 118-bus test systems. In addition to the traditional generating fuel cost, multi-fuels options, valve-point effect and other complexities have been considered. Furthermore, different objectives such as voltage profile improvement, voltage stability enhancement and emission reduction are considered. The obtained results are compared with those obtained using some well-known optimization algorithms. This comparison highlights the effectiveness of the BSA method for solving different OPF problems with complicated and non-smooth objective functions.

Introduction

There are three types of problems commonly referred to in power system literature: power flow, economic dispatch, and optimal power flow [1]. Economic dispatch (ED), applies several formulations to determine the least-cost generation dispatch to satisfy the total required demand, however these formulations simplify or even ignore power flow constraints [1]. The optimal power flow (OPF) has been initially proposed by French scholar Carpentier in 1962 [1]. Since then, it becomes one of the most important functions of operation, production, control and monitoring of power in modern energy systems [1,2].

A basic OPF problem seeks optimal distribution of the real power and/or the reactive power by adjusting the control variables, so that a specific objective in the operation of electric power system is optimized. During the optimization, the power flow balance, generator capability, transmission capability, voltage profile constraints must be satisfied.

Various traditional optimization techniques have been used to solve the OPF problem. These include linear programming (1979), Newton methods (1992), interior point methods (1998) and dynamic programming (2001). Pandya and Joshi in [3] and Frank et al. in [4] presented a comprehensive survey of various traditional optimization methods used to solve OPF problems. However, in practice, traditional methods suffer from some inadequacy. Some of their shortcomings among others are: firstly, they do not guarantee finding the global optimum, secondly, traditional methods involve complex calculations with long time, and they are not adapted for discrete variables [4].

Over the past few decades, many powerful metaheuristics have been developed. Some of them have been applied to the OPF problem with impressive success. Some of the recent applications of metaheuristics to OPF problem are: Artificial Bee Colony (ABC) [5,6], Black Hole (BH) [7], Teaching Learning Based Optimization (TLBO) [8], League Championship Algorithm (LCA) [9], Differential Search Algorithm (DSA) [10], Krill Herd Algorithm (KHA) [11], Gravitational Search Algorithm (GSA) [12,13], Imperialist Competitive Algorithm (ICA) [14] and Group Search Optimization (GSO) [15]. A review of many metaheuristics applied to solve the OPF problem is given in [16,17]. However, due to the variability of the objectives while solving OPF problems, no algorithm is the best in solving all OPF problems. Therefore, there is always a need for a new algorithm, which can efficiently solve the majority of the OPF problems.

The Backtracking Search Algorithm (BSA) which is developed by Civicioglu [18] is a new evolutionary algorithm (EA) developed to solve real-valued numerical optimization problems. As other EAs like genetic algorithm (GA) and differential evolution (DE), BSA is
The remainder of the paper is organized as follows. First, the problem formulation is presented in brief in section 2. Then, the main features of the BSA algorithm are presented in section 3. Next, the results after solving different cases of OPF problem using BSA are discussed in section 4. Finally, conclusions are drawn in the last section of this paper.

Problem formulation

The OPF is a power flow problem which gives the optimal settings of the control variables for a given settings of load by minimizing a predefined objective function such as the cost of active power generation or transmission losses. OPF considers the operating limits of the system and it can be formulated as a nonlinear constrained optimization problem as follows:

\[
\text{Minimize } J(x, u) \quad (1)
\]

\[
\text{Subject to } g(x, u) = 0 \quad (2)
\]

\[
\text{and } h(x, u) \leq 0 \quad (3)
\]

where \( u \) is the vector of independent variables or control variables, \( x \) is the vector of dependent variables or state variables, \( J(x, u) \) is the objective function, \( g(x, u) \) is the set of equality constraints and \( h(x, u) \) is the set of inequality constraints.

The control variables \( u \) and the state variables \( x \) of the OPF problem are defined as follows.

Control variables

These are the set of variables which can be adjusted to satisfy the load flow equations [9]. The set of control variables in the OPF problem formulation are:
active power generation at the PV buses except at the slack bus. \( P_{C_i} \)

- voltage magnitude at PV buses. \( V_{C_i} \)

- tap settings of transformer. \( T \)

- shunt VAR compensation. \( Q_C \)

Hence, \( u \) can be expressed as:
\[
u^* = \begin{bmatrix} P_{C_1}, \ldots, P_{C_{NG}}, \ V_{C_1}, \ldots, V_{C_{NG}}, \ Q_C, \ldots, Q_{C_{NG}}, \ T_1, \ldots, T_{NT} \end{bmatrix}
\] (4)

where \( NG, NT \) and \( NC \) are the number of generators, the number of regulating transformers and the number of VAR compensators, respectively.

State variables

These are the set of variables which describe any unique state of the system [9]. The set of state variables for the OPF problem formulation are:

- active power at slack bus. \( P_{Ci} \)

- voltage magnitude at PQ buses (load buses). \( V_l \)

- reactive power generation of all generator units. \( Q_G \)

- transmission line loading (or line flow). \( S_l \)

Hence, \( x \) can be expressed as:
\[
x^* = \begin{bmatrix} P_{Ci}, \ V_l, \ldots, V_{L_{NL}}, \ Q_{G_1}, \ldots, Q_{G_{NG}}, \ S_l, \ldots, S_{nl} \end{bmatrix}
\] (5)

where, \( NL \) and \( nl \) are the number of load buses and the number of transmission lines, respectively.

Constraints

OPF constraints can be classified into equality and inequality constraints, as detailed in the following sections.

Equality constraints

Set of constraints that embody the typical nonlinear power flow equations that control the power system, given as follows.

(a) Real power constraints:
\[
P_{Ci} - P_{Di} - \sum_{j=1}^{NB} V_j \left[ G_{ij} \cos (\theta_{ij}) + B_{ij} \sin (\theta_{ij}) \right] = 0
\] (6)

(b) Reactive power constraints:
\[
Q_{Ci} - Q_{Di} - \sum_{j=1}^{NB} V_j \left[ G_{ij} \sin (\theta_{ij}) - B_{ij} \cos (\theta_{ij}) \right] = 0
\] (7)

where \( \theta_{ij} = \theta_i - \theta_j \), \( NB \) is the number of buses, \( P_D \) is the active load demand, \( Q_G \) is the reactive power demand, \( G_{ij} \) and \( B_{ij} \) are the elements of the admittance matrix \( Y_{ij} = G_{ij} + jB_{ij} \) representing the conductance and susceptance between bus \( i \) and bus \( j \), respectively.

Inequality constraints

Set of constraints which reflect the system operational and the physical limits of the system given as follows.

(a) Generator constraints
For all generators including the slack: voltage, active and reactive outputs ought to be restricted by their lower and upper limits as follows:
\[
V_{G_i}^\text{min} \leq V_{G_i} \leq V_{G_i}^\text{max} \quad i = 1, \ldots, NG
\] (8)

\[
P_{G_i}^\text{min} \leq P_{G_i} \leq P_{G_i}^\text{max} \quad i = 1, \ldots, NG
\] (9)

\[
Q_{G_i}^\text{min} \leq Q_{G_i} \leq Q_{G_i}^\text{max} \quad i = 1, \ldots, NG
\] (10)

(b) Transformer constraints
Transformer tap settings must be restricted within their specified lower and upper limits as follows:
\[
T_i^\text{min} \leq T_i \leq T_i^\text{max} \quad i = 1, \ldots, NT
\] (11)

(c) Shunt VAR compensator constraints
Shunt VAR compensators must be bounded by their lower and upper limits as follows:
\[
Q_{Ci}^\text{min} \leq Q_C \leq Q_{Ci}^\text{max} \quad i = 1, \ldots, NC
\] (12)

(d) Security constraints
These constraints can be mathematically formulated as follows:
\[
V_{l_i}^\text{min} \leq V_{l_i} \leq V_{l_i}^\text{max} \quad i = 1, \ldots, NL
\] (13)

\[
S_{ni} \leq S_{ni}^\text{max} \quad i = 1, \ldots, nl
\] (14)

It is worth mentioning that control variables are self-constrained. The inequality constraints of dependent variables, which contain load bus voltage magnitude; real power generation at slack bus, reactive power generation and line loading can be included into the objective function as quadratic penalty terms. In these terms, a penalty factor multiplied with the square of the disregard value of dependent variable is added to the objective function and any unfeasible solution obtained is declined [12]. Details about how penalty factors can be selected are given in [23]. Mathematically, penalty function can be expressed as follows:
\[
\text{Penalty} = \lambda_P \left( P_{Ci} - P_{lim} \right)^2 + \lambda_V \sum_{i=1}^{NL} \left( V_{l_i} - V_{lim} \right)^2
\]
\[
+ \lambda_Q \sum_{i=1}^{NG} \left( Q_{Ci} - Q_{lim} \right)^2 + \lambda_S \sum_{i=1}^{nl} \left( S_{ni} - S_{ni}^\text{max} \right)^2
\]
(15)

where \( \lambda_P, \lambda_V, \lambda_Q \) and \( \lambda_S \) are penalty factors and \( x^\text{lim} \) is the violated limit value of the dependent variable \( x \) and can be defined as follows:
\[
x^\text{lim} = \begin{cases} x^\text{max} & x > x^\text{max} \\ x^\text{min} & x < x^\text{min} \end{cases}
\]
(16)

Objective function

Several objective functions are considered in this paper which are: cost reduction (with and without multi-fuels options or valve-point effect), voltage profile improvement, voltage stability enhancement and emission reduction. Moreover, some combinations between these objective functions are considered also.

Backtracking Search Algorithm (BSA)

As previously mentioned, BSA is a new EA developed by Civicioglu for solving real-valued numerical optimization problems [18]. It has only one tuning parameter and it uses the three basic operators that are selection, mutation and crossover. The general structure of the BSA is given in Algorithm 1 shown below. From this algorithm, it can be noticed that the optimization process of
BSA is achieved following five steps namely: initialization, selection-I, mutation, crossover, and selection-II. The following subsections describe in details these steps.

Algorithm 1. General structure of BSA [18].

1. Initialization
   repeat
   2. Selection-I
   3. Generation of Trial-Population
   4. Crossover
   5. Selection-II
   until stopping conditions are met

Initialization

As many other metaheuristics, in the initialization step the BSA generates a random initial population $P$ of $N$ individuals in the search space that has $D$ dimensions as shown in lines (3–9) of Algorithm 2.

Selection-I

In this second step, a historical population called $oldP$ is determined for calculation of the search direction. $oldP$ is initialized like the population $P$ as shown in line (6) of Algorithm 2. However, $oldP$ can be redefined in every iteration using an ‘if-then’ rule as shown in line (11) of Algorithm 2. After $oldP$ is determined, the order of individuals inside it is randomly permutated (line (12) of Algorithm 2).

Mutation

In the mutation step, a mutant is generated as the initial form of trial population ($T$) using the old population ($oldP$) and the population ($P$) itself as follows [18]:

$$\text{Mutant} = P + F \times (oldP - P)$$

where $F$ is the scale factor used to control the search direction matrix ($oldP$-$P$). It is worth to mention here that using the old population in mutation allows BSA to benefit from the experience of previous generations [18]. The mutation step is shown in line 14 in Algorithm 2.

Crossover

The crossover operator used in BSA is different from the one used in DE or its variants. The crossover process represents the last step while generating the trial population. This process starts by generating a binary integer-valued matrix of size $(N \times D)$ denoted $\text{map}_{i,j}$ using two strategies as shown in Algorithm 2 (lines from (15) through (22)). This matrix indicates the individuals of $T$ to be manipulated using the relevant individuals of $P$ as shown in line (26) of Algorithm 2 [18].

Selection II

In this step, the trial population $T$ is used to update the population $P$ in a greedy manner as shown in Algorithm 2 (lines from (37) through (42)). If two individuals have the same order, one from the trial population $T_i$ and one from the population $P_i$, and $T_i$ has a better fitness than $P_i$, therefore, $P_i$ is replaced by $T_i$ [18].

Algorithm 2. Pseudocode of BSA [18].

1 function bsa (ObjFun, N, D, maxcycle, low, up)
   Input: ObjFun, N, D, maxcycle, mixrate, low$_1$: D, up$_1$: D
   Output: globalminimum, globalminimizer

// INITIALIZATION
2 globalminimum = inf
3 for i from 1 to N do
4     $P_i$ := ObjFun ($P_i$) // Initial-fitness values of $P$.
5 end
6 for iteration from 1 to maxcycle do
7     // SELECTION-I
8     if ($a < b$, $a \sim U(0,1)$) then oldP := $P$ end
9     oldP := permuting (oldP)
10 Generation of Trial-Population
11     mutable = $P + F \times (oldP - P)$
12 // MUTATION
13     for i from 1 to N do
14         for j from 1 to D do
15             $P_{i,j}$ := $P_{i,j}$ // Initial-map is an N-by-D matrix of ones.
16             if ($c < d$, $c \sim U(0,1)$) then
17                 for i from 1 to N do
18                     $map_{i,j}$ := permuting ($[1, 2, 3, \ldots, D]$)
19                     for j from 1 to D do
20                         for i from 1 to N do
21                             if $map_{i,j}$ = 1 then $T_{i,j}$ := $P_{i,j}$ end
22                     end
23                     end
24                     $T$ := mutant
25                     for i from 1 to N do
26                         $map_{i,j}$ := permuting ($[1, 2, 3, \ldots, D]$)
27                     end
28                     end
29                     // Boundary Control Mechanism
30                     for i from 1 to N do
31                         for j from 1 to D do
32                             if ($T_{i,j} < \text{low}_j$) or ($T_{i,j} > \text{up}_j$) then
33                                 $T_{i,j}$ := $P_{i,j} - \text{low}_j$ + $low_j$
34                             end
35                         end
36                     end
37                     // SELECTION-II
38                     for i from 1 to N do
39                         if fitness $T_i$ < fitness $P_i$ then
40                             fitness $P_i$ := fitness $T_i$
41                             $P_i$ := $T_i$
42                         end
43                     end
44                     if fitness $P_{best} < \text{globalminimum}$ then
45                         globalminimum := fitness $P_{best}$
46                         globalminimizer := $P_{best}$
47                     end
48                     for i from 1 to N do
49                         for j from 1 to D do
50                             if fitness $P_{i,j}$ < fitness $T_{i,j}$ then
51                                 $P_{i,j}$ := $T_{i,j}$
52                             end
53                         end
54                     end
55                     end
56 end
Application and results

It is worth mentioning that, the developed software program is written in the commercial MATLAB computing environment using some MATPOWER programs. It is applied on a 2.20 GHz i7 personal computer with 8.00 GB-RAM and using parallel processing to run the different runs.

In this paper, the proposed BSA algorithm has been applied on three test systems that are the IEEE 30-bus, the IEEE 57-bus, and IEEE-118 bus test systems. Moreover, 16 different case studies are investigated. These cases start from basic cases then different complexities are included and other objectives are considered. These cases are summarized in Table 1. In our implementation, the transformer tap settings have been considered as discrete variable with a step of 0.125 p.u.

IEEE 30-bus test system

The IEEE 30-bus test system has a total generation capacity of 435.0 MW and its main characteristics are given in Table 2. Detailed data can be derived from [24]. Cost and emission coefficients used in this paper are given in Table 3. Table 4 gives cost coefficients when considering multi-fuels options for generators 1 and 2.

CASE 1: OPF by considering cost reduction

The first case investigated in this paper is the base case, which consists of minimizing the generation fuel cost expressed by a quadratic function. Hence, the objective function (or more precisely the augmented objective function) for this case is:

\[
J(x, u) = \left( \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \right) + \text{Penalty}
\]  

where \(a_i, b_i\) and \(c_i\) are the cost coefficients of the \(i\)th generator.

The proposed BSA technique has been run for this case and the optimal control variables obtained after optimization are tabulated in Table 5 where the optimal generation fuel cost obtained is (799.0760 $/h). Furthermore, the evolution of the objective function over iterations is sketched in Fig. 1. This figure shows the evolution of cost (without penalty) and the penalty term over iterations. It can be seen that the penalty term tends to zero after 34 iterations only. In other words, the BSA finds a feasible solution for this case after 34 iterations.

CASE 2: OPF by considering cost reduction and voltage profile improvement

Bus voltage is one of the most important and significant indications of safety and service quality of power systems. The solution found in CASE 1 is a purely cost based objective function. Such solution may have an undesirable voltage profile [25]. Thus, this example aims at minimizing fuel cost with a flatter voltage profile by considering a twofold objective function [26]. The voltage profile is optimized by minimizing the load bus voltage deviation (\(VD\)) from 1.0 p.u, which is given by:

\[
VD = \sum_{j=1}^{NL} |V_{Li} - 1|
\]  

Therefore, the objective function can be expressed as:

\[
J(x, u) = \left( \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \right) + \lambda_{VD}(VD) + \text{Penalty}
\]  

Table 2

The main characteristics of the IEEE 30-bus test system.

<table>
<thead>
<tr>
<th>System characteristics</th>
<th>Value</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buses</td>
<td>30</td>
<td>–</td>
</tr>
<tr>
<td>Branches</td>
<td>41</td>
<td>–</td>
</tr>
<tr>
<td>Generators</td>
<td>6</td>
<td>Buses: 1, 2, 5, 8, 11 and 13</td>
</tr>
<tr>
<td>Shunts</td>
<td>9</td>
<td>Branches: 10, 12, 15, 17, 20, 21, 23, 24 and 29</td>
</tr>
<tr>
<td>Transformers</td>
<td>4</td>
<td>Branches: 11, 12, 15 and 36</td>
</tr>
<tr>
<td>Control variables</td>
<td>24</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 1

Different case studies investigated in this paper.

<table>
<thead>
<tr>
<th>Name</th>
<th>Cost</th>
<th>Voltage profile</th>
<th>Voltage stability</th>
<th>Multi-fuels</th>
<th>Valve-point effect</th>
<th>Emission</th>
<th>System</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE 1</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 2</td>
<td>✔</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 3</td>
<td>✔</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 4</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 5</td>
<td>✔</td>
<td>✔</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 6</td>
<td>✔</td>
<td>-</td>
<td>✔</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 7</td>
<td>✔</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 8</td>
<td>✔</td>
<td>✔</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 9</td>
<td>✔</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 10</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 11</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✔</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 12</td>
<td>✔</td>
<td>✔</td>
<td>-</td>
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<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 13</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 14</td>
<td>✔</td>
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<td>✔</td>
<td>-</td>
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<td>-</td>
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</tr>
<tr>
<td>CASE 15</td>
<td>✔</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✔</td>
<td>-</td>
<td>IEEE 30-bus test system</td>
</tr>
<tr>
<td>CASE 16</td>
<td>✔</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>IEEE 30-bus test system</td>
</tr>
</tbody>
</table>
Table 3  
Cost and emission coefficients for the IEEE 30-bus test system.

<table>
<thead>
<tr>
<th>Generator</th>
<th>Bus</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>x</th>
<th>β</th>
<th>γ</th>
<th>α</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0.00375</td>
<td>18</td>
<td>0.037</td>
<td>4.091</td>
<td>–5.554</td>
<td>6.49</td>
<td>2.00e-04</td>
<td>2.857</td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>0</td>
<td>1.75</td>
<td>0.0175</td>
<td>16</td>
<td>0.038</td>
<td>2.543</td>
<td>–6.047</td>
<td>5.638</td>
<td>5.00e-04</td>
<td>3.333</td>
</tr>
<tr>
<td>G3</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>0.0625</td>
<td>14</td>
<td>0.04</td>
<td>4.258</td>
<td>–5.094</td>
<td>4.586</td>
<td>1.00e-06</td>
<td>8</td>
</tr>
<tr>
<td>G4</td>
<td>8</td>
<td>0</td>
<td>3.25</td>
<td>0.00834</td>
<td>12</td>
<td>0.045</td>
<td>5.326</td>
<td>–3.55</td>
<td>3.38</td>
<td>2.00e-03</td>
<td>2</td>
</tr>
<tr>
<td>G5</td>
<td>11</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>13</td>
<td>0.042</td>
<td>4.258</td>
<td>–5.094</td>
<td>4.586</td>
<td>1.00e-06</td>
<td>8</td>
</tr>
<tr>
<td>G6</td>
<td>13</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>13.5</td>
<td>0.041</td>
<td>6.131</td>
<td>–5.555</td>
<td>5.151</td>
<td>1.00e-05</td>
<td>6.667</td>
</tr>
</tbody>
</table>

Table 4  
Cost coefficients for generator 1 and 2 of the IEEE 30-bus test system for multi-fuels sources.

<table>
<thead>
<tr>
<th>Generator</th>
<th>Bus</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>P_max</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>P_max</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>55</td>
<td>0.7</td>
<td>0.005</td>
<td>50</td>
<td>140</td>
<td>82.5</td>
<td>1.05</td>
<td>0.0075</td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>40</td>
<td>0.3</td>
<td>0.01</td>
<td>20</td>
<td>55</td>
<td>80</td>
<td>0.6</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 5  
Optimal settings of control variables found for CASE 1 through CASE 10.

<table>
<thead>
<tr>
<th>CASE 1</th>
<th>CASE 2</th>
<th>CASE 3</th>
<th>CASE 4</th>
<th>CASE 5</th>
<th>CASE 6</th>
<th>CASE 7</th>
<th>CASE 8</th>
<th>CASE 9</th>
<th>CASE 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_{k1}</td>
<td>177.3838</td>
<td>173.1714</td>
<td>172.0277</td>
<td>139.9204</td>
<td>139.5920</td>
<td>139.7202</td>
<td>198.7223</td>
<td>197.0323</td>
<td>197.6731</td>
</tr>
<tr>
<td>P_{k2}</td>
<td>48.8335</td>
<td>48.3903</td>
<td>48.8610</td>
<td>54.9886</td>
<td>54.2062</td>
<td>54.7933</td>
<td>44.3031</td>
<td>41.8254</td>
<td>46.0134</td>
</tr>
<tr>
<td>V_q</td>
<td>1.0000</td>
<td>1.0441</td>
<td>1.0880</td>
<td>1.0863</td>
<td>1.0356</td>
<td>1.0902</td>
<td>1.1000</td>
<td>1.0514</td>
<td>1.0954</td>
</tr>
<tr>
<td>T_{l1(1-9)}</td>
<td>1.0250</td>
<td>1.0500</td>
<td>0.9750</td>
<td>1.0625</td>
<td>1.0375</td>
<td>0.9625</td>
<td>1.0000</td>
<td>1.0250</td>
<td>0.9750</td>
</tr>
<tr>
<td>T_{l2(10-27)}</td>
<td>0.9000</td>
<td>0.9000</td>
<td>0.9000</td>
<td>0.9125</td>
<td>0.9000</td>
<td>0.9250</td>
<td>1.0125</td>
<td>0.9125</td>
<td>0.9000</td>
</tr>
<tr>
<td>T_{l3(28-40)}</td>
<td>0.9625</td>
<td>0.9625</td>
<td>0.9500</td>
<td>0.9875</td>
<td>0.9625</td>
<td>0.9500</td>
<td>1.0000</td>
<td>0.9750</td>
<td>0.9375</td>
</tr>
<tr>
<td>P_{C1}</td>
<td>4.2998</td>
<td>5.0000</td>
<td>4.5315</td>
<td>5.0000</td>
<td>4.3687</td>
<td>4.4714</td>
<td>4.3411</td>
<td>3.8622</td>
<td>4.5090</td>
</tr>
<tr>
<td>P_{C2}</td>
<td>4.6378</td>
<td>7.2431</td>
<td>4.5338</td>
<td>5.0000</td>
<td>4.6103</td>
<td>4.9527</td>
<td>4.9742</td>
<td>4.5177</td>
<td>4.5129</td>
</tr>
<tr>
<td>P_{C3}</td>
<td>4.9016</td>
<td>3.7630</td>
<td>4.6750</td>
<td>5.0000</td>
<td>3.3418</td>
<td>5.0000</td>
<td>4.2538</td>
<td>2.9673</td>
<td>4.4122</td>
</tr>
<tr>
<td>P_{C4}</td>
<td>5.0000</td>
<td>2.3539</td>
<td>4.2197</td>
<td>5.0000</td>
<td>5.0000</td>
<td>5.0000</td>
<td>4.7605</td>
<td>5.0000</td>
<td>4.9965</td>
</tr>
<tr>
<td>P_{C5}</td>
<td>4.0889</td>
<td>4.9912</td>
<td>5.1123</td>
<td>5.0000</td>
<td>4.9360</td>
<td>4.0597</td>
<td>5.0000</td>
<td>5.0000</td>
<td>3.9809</td>
</tr>
<tr>
<td>P_{C6}</td>
<td>5.0000</td>
<td>3.6589</td>
<td>5.0000</td>
<td>5.0000</td>
<td>5.0000</td>
<td>5.0000</td>
<td>4.3837</td>
<td>5.4959</td>
<td>5.0000</td>
</tr>
<tr>
<td>P_{C8}</td>
<td>4.8423</td>
<td>4.8500</td>
<td>3.6102</td>
<td>5.0000</td>
<td>5.0000</td>
<td>4.3471</td>
<td>5.0000</td>
<td>5.0000</td>
<td>4.9781</td>
</tr>
<tr>
<td>P_{C10}</td>
<td>0.9760</td>
<td>803.4294</td>
<td>800.3340</td>
<td>646.1504</td>
<td>653.1019</td>
<td>648.1425</td>
<td>830.7779</td>
<td>836.8811</td>
<td>832.7029</td>
</tr>
<tr>
<td>Emission (ton/h)</td>
<td>0.3761</td>
<td>0.3546</td>
<td>0.3514</td>
<td>0.2833</td>
<td>0.2805</td>
<td>0.2821</td>
<td>0.4377</td>
<td>0.4330</td>
<td>0.4343</td>
</tr>
</tbody>
</table>

where $\lambda_{VD}$ is a weighting factor which has to be selected carefully in order to give the desired amount of weight or importance of the VD term compared with the cost term. After many tests, $\lambda_{VD}$ is selected as 1000 in this study to balance both objectives.

The proposed BSA technique has been run for this case and the optimal control variables obtained are given in Table 5. The first comment that can be concluded from this table is that $\lambda_{VD}$ has been operated a voltage stability index called $\lambda_{VD}$.

CASE 3: OPF by considering cost reduction and voltage stability enhancement

Prediction of voltage instability is an issue of paramount importance in power systems. In [27] Kessel and Glavitch have developed a voltage stability index called $l_{max}$ which is defined based on local indicators $l_{max}$ and it is given by:

![Fig. 1. Objective function variation for CASE 1.](image-url)

Fig. 1. Objective function variation for CASE 1.
where $H$ matrix is generated by the partial inversion of $Y_{bus}$. More details can be found in [27].

The indicator $L_{max}$ varies between 0 and 1 where the lower the indicator, the more the system stable. Therefore, enhancing voltage stability can be achieved through the minimization of $L_{max}$ of the whole system [28]. Thus, the objective function in this case is:

$$J(x,u) = \sum_{i=1}^{NC} a_i + b_iP_{G_i} + c_iP_{G_i}^2 + \lambda_{L_{max}}(L_{max}) + \text{Penalty}$$

where $\lambda_{L_{max}}$ is a weighting factor selected as 6000 in this study.

The results of the optimization study are given in Table 5 while the trend of convergence is shown in Fig. 4. It appears that the $L_{max}$ has been reduced from 0.1273 to 0.1259 compared with CASE 1.

**CASE 4: OPF by considering multi-fuels**

In some practical cases, thermal generating plants may have multi-fuel sources like coal, natural gas and oil. Hence, the fuel cost curve can be expressed by a piecewise quadratic functions as follows [28]:

$$f_i = a_k + b_kP_{G_i} + c_kP_{G_i}^2 \quad \text{if} \quad P_{G_i}^{\text{min}} \leq P_{G_i} \leq P_{G_i}^{\text{min}}$$

where $k$ is the fuel option. In this study, generators 1 and 2 have two fuel options ($k = 2$). The generator fuel cost coefficients are given in Table 4. The objective function of CASE 4 is given by:
The main characteristics of the IEEE 57-bus test system.

<table>
<thead>
<tr>
<th>System Characteristics</th>
<th>Value</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buses</td>
<td>57</td>
<td>[33]</td>
</tr>
<tr>
<td>Branches</td>
<td>80</td>
<td>[33]</td>
</tr>
<tr>
<td>Generators</td>
<td>7</td>
<td>Buses: 1, 2, 3, 6, 8, 9 and 12</td>
</tr>
<tr>
<td>Shunts</td>
<td>3</td>
<td>Buses: 18, 25 and 53</td>
</tr>
<tr>
<td>Transformers</td>
<td>17</td>
<td>Branches: 19, 20, 31, 35, 36, 37, 41, 46, 54, 58, 59, 65, 66, 71, 73, 76 and 80</td>
</tr>
<tr>
<td>Control variables</td>
<td>33</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 7
Cost and emission coefficients for the IEEE 57-bus test system.

<table>
<thead>
<tr>
<th>Generator</th>
<th>Bus</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>x</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \alpha )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0.00375</td>
<td>18</td>
<td>0.037</td>
<td>4.091</td>
<td>-5.554</td>
<td>6.49</td>
<td>2.00E-04</td>
<td>2.86E-01</td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>0</td>
<td>1.75</td>
<td>0.0175</td>
<td>16</td>
<td>0.038</td>
<td>2.543</td>
<td>-6.047</td>
<td>5.638</td>
<td>5.00E-04</td>
<td>3.33E-01</td>
</tr>
<tr>
<td>G3</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>13.5</td>
<td>0.041</td>
<td>6.131</td>
<td>-5.555</td>
<td>5.151</td>
<td>1.00E-05</td>
<td>6.67E-01</td>
</tr>
<tr>
<td>G4</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>0.00375</td>
<td>18</td>
<td>0.037</td>
<td>3.491</td>
<td>-5.754</td>
<td>6.39</td>
<td>3.00E-04</td>
<td>2.66E-01</td>
</tr>
<tr>
<td>G5</td>
<td>5</td>
<td>8</td>
<td>1</td>
<td>0.0625</td>
<td>14</td>
<td>0.04</td>
<td>4.258</td>
<td>-5.094</td>
<td>4.586</td>
<td>1.00E-06</td>
<td>8.00E-01</td>
</tr>
<tr>
<td>G6</td>
<td>6</td>
<td>9</td>
<td>0.75</td>
<td>0.0195</td>
<td>15</td>
<td>0.039</td>
<td>2.754</td>
<td>-5.847</td>
<td>5.238</td>
<td>4.00E-04</td>
<td>2.88E-01</td>
</tr>
<tr>
<td>G7</td>
<td>7</td>
<td>12</td>
<td>0</td>
<td>3.25</td>
<td>0.00834</td>
<td>12</td>
<td>0.045</td>
<td>5.326</td>
<td>-3.555</td>
<td>3.38</td>
<td>2.00E-03</td>
</tr>
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</table>

Table 8
Optimal settings of control variables found for CASE 11 through CASE 15.

<table>
<thead>
<tr>
<th>Generator</th>
<th>Bus</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>x</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \alpha )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0.00375</td>
<td>18</td>
<td>0.037</td>
<td>4.091</td>
<td>-5.554</td>
<td>6.49</td>
<td>2.00E-04</td>
<td>2.86E-01</td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>0</td>
<td>1.75</td>
<td>0.0175</td>
<td>16</td>
<td>0.038</td>
<td>2.543</td>
<td>-6.047</td>
<td>5.638</td>
<td>5.00E-04</td>
<td>3.33E-01</td>
</tr>
<tr>
<td>G3</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>13.5</td>
<td>0.041</td>
<td>6.131</td>
<td>-5.555</td>
<td>5.151</td>
<td>1.00E-05</td>
<td>6.67E-01</td>
</tr>
<tr>
<td>G4</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>0.00375</td>
<td>18</td>
<td>0.037</td>
<td>3.491</td>
<td>-5.754</td>
<td>6.39</td>
<td>3.00E-04</td>
<td>2.66E-01</td>
</tr>
<tr>
<td>G5</td>
<td>5</td>
<td>8</td>
<td>1</td>
<td>0.0625</td>
<td>14</td>
<td>0.04</td>
<td>4.258</td>
<td>-5.094</td>
<td>4.586</td>
<td>1.00E-06</td>
<td>8.00E-01</td>
</tr>
<tr>
<td>G6</td>
<td>6</td>
<td>9</td>
<td>0.75</td>
<td>0.0195</td>
<td>15</td>
<td>0.039</td>
<td>2.754</td>
<td>-5.847</td>
<td>5.238</td>
<td>4.00E-04</td>
<td>2.88E-01</td>
</tr>
<tr>
<td>G7</td>
<td>7</td>
<td>12</td>
<td>0</td>
<td>3.25</td>
<td>0.00834</td>
<td>12</td>
<td>0.045</td>
<td>5.326</td>
<td>-3.555</td>
<td>3.38</td>
<td>2.00E-03</td>
</tr>
</tbody>
</table>

\[ \text{Emission} = \sum_{i=1}^{NG} 10^{-2} \left( a_i + b_i P_{Gi} + c_i P_{Gi}^2 \right) + \left( \alpha Xi(P_{Gi}) \right) \]  

where \( a_i, b_i, c_i, \alpha_i \) and \( \mu_i \) are coefficients of the ith generator emission characteristics.

Therefore, the objective function for this case can be expressed by:

\[ J(x, u) = \left( \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \right) + \lambda_{\text{Emission}(\text{Emission})} + \text{Penalty} \]  

where \( \lambda_{\text{Emission}} \) is a weighting factor and it is equal to 1000 in this case.

The results obtained after optimization using the BSA algorithm are displayed in Table 5 and the trend of optimization is represented in Fig. 5. The results show that the emission has been reduced from (0.3671 ton/h) to (0.2425 ton/h) i.e. by 33.94%, however, the total fuel cost has increased from (799.0760 $/h) to (835.0199 $/h) i.e. by 4.50% compared with CASE 1.

IEEE 57-bus test system

In order to test the scalability of the proposed BSA algorithm, a larger test system is considered in this paper, which is the IEEE 57-bus test system. This system has a total generation capacity of 1975.9 MW and its main characteristics are given in Table 6. Cost and emission coefficients for this system are given in Table 7. Detailed data can be derived from [33].
CASE 11: OPF by considering cost reduction

The aim here is to minimize the total generating fuel cost. Therefore, the objective function of this case is given by (18). The BSA is run in order to find the optimal settings for CASE 11 and the obtained results are given in Table 8. The cost obtained for this case is (6411.0043 $/h) while $V_D$ and $L_{\text{max}}$ are (1.1009 p.u.) and (0.2819), respectively.

CASE 12: OPF by considering cost reduction and voltage profile improvement

As in CASE 2, the goal here is to optimize both the total generating fuel cost and the voltage profile. Hence the objective function is given by (20) where $\lambda_{VD} = 10.000$ in this case to balance between the objectives. The results of such optimization using the proposed BSA algorithm are tabulated in Table 8. It appears from this table that the $V_D$ has been reduced from (1.1009 p.u.) to (0.6888 p.u.) compared with CASE 11. However, the cost has slightly increased from (6411.0043 $/h) to (6436.7551 $/h) compared with CASE 11. The improvement of the voltage profile is illustrated in Fig. 6.

CASE 13: OPF by considering cost reduction and voltage stability enhancement

The voltage stability enhancement has been also tested for the IEEE 57-test system in this case where the aim is to minimize the cost and at the same time to improve the vulnerability of the system by enhancing its voltage stability. Thus, the objective function of CASE 13 is given by (23) with $\lambda_{L_{\text{max}}} = 10.000$. The BSA is run for this case and the obtained control variables are tabulated in Table 8. These results show clearly that the voltage stability of the system is enhanced with a slight augmentation in cost.

CASE 14: OPF by considering valve-point effect

The effect of valve-point loading is tested in this case. Therefore, the objective function of this case is given by (28). The obtained results using BSA are given in Table 8. In this case the cost has marginally augmented from (6411.0043 $/h) to (6462.4093 $/h) compared with CASE 11.

Table 9

The main characteristics of the IEEE 118-bus test system.

<table>
<thead>
<tr>
<th>System characteristics</th>
<th>Value</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buses</td>
<td>118</td>
<td>[33]</td>
</tr>
<tr>
<td>Branches</td>
<td>186</td>
<td>[33]</td>
</tr>
<tr>
<td>Generators</td>
<td>54</td>
<td>Buses: 1, 4, 6, 8, 10, 12, 15, 18, 19, 24, 25, 26, 27, 31, 32, 34, 36, 40, 42, 46, 49, 54, 55, 56, 59, 61, 62, 65, 66, 69, 70, 72, 73, 74, 76, 77, 80, 85, 87, 89, 90, 91, 92, 99, 100, 103, 104, 105, 107, 111, 112, 113 and 116</td>
</tr>
<tr>
<td>Shunts</td>
<td>14</td>
<td>Buses: 5, 34, 37, 44, 45, 46, 48, 74, 79, 82, 83, 105, 107 and 110</td>
</tr>
<tr>
<td>Transformers</td>
<td>9</td>
<td>Branches: 8, 32, 36, 51, 93, 95, 102, 107 and 127</td>
</tr>
<tr>
<td>Control variables</td>
<td>130</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 6. Comparison of voltage profile between (a) CASE 11 and (b) CASE 12.
Table 10
Optimal settings of control variables found for CASE 16.

<table>
<thead>
<tr>
<th>PG</th>
<th>PG</th>
<th>PG</th>
<th>PG</th>
<th>PG</th>
<th>PG</th>
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<th>PG</th>
<th>PG</th>
<th>PG</th>
<th>PG</th>
<th>PG</th>
</tr>
</thead>
<tbody>
<tr>
<td>220.6014</td>
<td>30.2083</td>
<td>32.1285</td>
<td>30.1760</td>
<td>30.4660</td>
<td>30.2968</td>
<td>30.2084</td>
<td>30.8136</td>
<td>35.8272</td>
<td>166.5605</td>
<td>47.4169</td>
<td>30.6967</td>
</tr>
<tr>
<td>119.9260</td>
<td>124.4234</td>
<td>30.5199</td>
<td>285.5017</td>
<td>289.5617</td>
<td>30.2547</td>
<td>30.5085</td>
<td>30.5679</td>
<td>30.0000</td>
<td>34.7917</td>
<td>30.4234</td>
<td>342.1612</td>
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<td>30.7890</td>
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<td>30.0000</td>
<td>30.2901</td>
<td>30.3413</td>
<td>0.9787</td>
<td>1.0046</td>
<td>0.9955</td>
<td>1.0034</td>
<td>1.0199</td>
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<td>0.9942</td>
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</table>

Table 10 (continued)

<table>
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<th>PG</th>
<th>PG</th>
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<th>PG</th>
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<th>PG</th>
<th>PG</th>
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</thead>
<tbody>
<tr>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
<td>VG</td>
</tr>
<tr>
<td>1.0087</td>
<td>1.0038</td>
<td>0.9989</td>
<td>0.9998</td>
<td>1.0064</td>
<td>1.0037</td>
<td>0.9993</td>
<td>1.0114</td>
<td>1.0187</td>
<td>1.0518</td>
<td>1.0131</td>
<td>1.0029</td>
</tr>
<tr>
<td>0.9945</td>
<td>0.9808</td>
<td>1.0045</td>
<td>1.0133</td>
<td>0.9932</td>
<td>1.0099</td>
<td>1.0025</td>
<td>0.9940</td>
<td>1.0070</td>
<td>0.9974</td>
<td>1.0108</td>
<td>1.0099</td>
</tr>
<tr>
<td>1.0126</td>
<td>0.9883</td>
<td>1.0009</td>
<td>0.9988</td>
<td>0.9750</td>
<td>1.0125</td>
<td>1.0000</td>
<td>0.9875</td>
<td>1.0000</td>
<td>0.9875</td>
<td>1.0000</td>
<td>0.9875</td>
</tr>
<tr>
<td>1.3480</td>
<td>2.4031</td>
<td>1.6355</td>
<td>2.6899</td>
<td>5.0000</td>
<td>3.6900</td>
<td>3.2356</td>
<td>4.9106</td>
<td>4.9851</td>
<td>3.1106</td>
<td>3.4727</td>
<td>0.0451</td>
</tr>
<tr>
<td>3.2741</td>
<td>2.8543</td>
<td>135333.4743</td>
<td>0.6773</td>
<td>0.0698</td>
<td>64.4897</td>
<td>384.8242</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CASE 15: OPF by considering emission

This case is similar to CASE 10 where the aim is to reduce the cost while monitoring the emission at the same time. The objective function is given by (30) with $\delta_{\text{emission}} = 1000$. The BSA is run for this case, the obtained control variables are tabulated in Table 8. It appears that, the emission has been reduced by 35.13% i.e. from (1.9726 ton/h) to (1.2796 ton/h) in this case, whilst the cost has slightly augmented from (6411.0043 $/h) to (6652.9484 $/h) i.e. by 3.77%.
IEEE 118-bus test system

A large-scale test system is considered in this paper, which is the IEEE 118-bus test system. This system has a total generation capacity of 9966.2 MW and its main characteristics are given in Table 9 and cost coefficients are given in [33]. Detailed data can be derived from [33].

CASE 16: OPF by considering cost reduction

The aim here is to minimize the total generating fuel cost. Therefore, the objective function of this system is given by (18). The BSA is run in order to find the optimal settings for CASE 16 and the obtained results are given in Table 10. We can notice from this table that, the minimum cost obtained is (135333.4743 $/h).

Performance evaluation study

In order to evaluate the performance of the BSA, it has been compared to several optimization algorithms including DE, PSO, GA, ABC and BBO. Control parameters of optimization algorithms used in this investigation are given in Table 11.

In Table 12 a statistical analysis of the results is tabulated. In each cell that corresponds to the performance of an algorithm for one specific case, we give five values that are: the min (or best), the mean (or average), the median, the max (or worst) and the standard deviation values. Furthermore, in the last row of Table 12, we calculated the feasibility rate (FR) which is defined as the number of runs where the algorithm converges to a feasible solution over the number of all runs. The FR is computed in this paper for all cases at the same time therefore the total number of runs is 16 (cases) \times 30 (runs per case) which equals 480.

From this table we can notice that the BSA is very robust in solving several OPF problems with different complexities. The averages values are very near to the best ones for all cases. The maximum average value with respect to the best one is obtained for CASE 5. The normalized error between these two values is 1.48%. Moreover, the BSA is the only algorithm that has a FR of 100% compared with the remaining algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Control parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSA</td>
<td>F = 1\times\text{normrnd}(0.3)</td>
<td>Scale factor (where normrnd(mean, standard deviation) generates random numbers from the normal distribution with mean and standard deviation parameters)</td>
</tr>
<tr>
<td></td>
<td>$F_{\text{generation's strategy}}$</td>
<td>Pseudo-stable walk (levy-like)</td>
</tr>
<tr>
<td>DE</td>
<td>$F = 0.8$</td>
<td>Differential weight</td>
</tr>
<tr>
<td></td>
<td>$C_r = 0.8905$</td>
<td>Crossover probability</td>
</tr>
<tr>
<td>PSO</td>
<td>$w = 0.5$</td>
<td>Inertia factor</td>
</tr>
<tr>
<td></td>
<td>$c_1 = 1.5$</td>
<td>Cognitive factor</td>
</tr>
<tr>
<td></td>
<td>$c_2 = 1.5$</td>
<td>Social factor</td>
</tr>
<tr>
<td>GA</td>
<td>$\text{Mutrate} = 0.5$</td>
<td>Mutation rate</td>
</tr>
<tr>
<td></td>
<td>$\text{Selection} = 0.75$</td>
<td>Fraction of population kept</td>
</tr>
<tr>
<td>ABC</td>
<td>$\text{FoodNumber} = \frac{N_p}{2}$</td>
<td>Number of food sources</td>
</tr>
<tr>
<td>BBO</td>
<td>$\text{pmutate} = 0$</td>
<td>Mutation probability</td>
</tr>
<tr>
<td></td>
<td>$\text{pmodify} = 1$</td>
<td>Habitat modification probability</td>
</tr>
<tr>
<td></td>
<td>$\text{pmutate} = 0.005$</td>
<td>Initial mutation probability</td>
</tr>
</tbody>
</table>

Common parameters

- Population size = 50 for cases: 1, 2, 3, 7, 8, 9 and 10
- Population size = 90 for cases: 4, 5, 6, 11, 12, 13, 14, 15 and 16
- Maximum number of iterations = 500 for cases: 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10
- Maximum number of iterations = 1500 for cases: 11, 12, 13, 14 and 15
- Maximum number of iterations = 2500 for case: 16

### Table 11
Control parameters of the related algorithms used in the tests.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Control parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSA</td>
<td>$F = 1\times\text{normrnd}(0.3)$</td>
<td>Scale factor (where normrnd(mean, standard deviation) generates random numbers from the normal distribution with mean and standard deviation parameters)</td>
</tr>
<tr>
<td></td>
<td>$F_{\text{generation's strategy}}$</td>
<td>Pseudo-stable walk (levy-like)</td>
</tr>
<tr>
<td>DE</td>
<td>$F = 0.8$</td>
<td>Differential weight</td>
</tr>
<tr>
<td></td>
<td>$C_r = 0.8905$</td>
<td>Crossover probability</td>
</tr>
<tr>
<td>PSO</td>
<td>$w = 0.5$</td>
<td>Inertia factor</td>
</tr>
<tr>
<td></td>
<td>$c_1 = 1.5$</td>
<td>Cognitive factor</td>
</tr>
<tr>
<td></td>
<td>$c_2 = 1.5$</td>
<td>Social factor</td>
</tr>
<tr>
<td>GA</td>
<td>$\text{Mutrate} = 0.5$</td>
<td>Mutation rate</td>
</tr>
<tr>
<td></td>
<td>$\text{Selection} = 0.75$</td>
<td>Fraction of population kept</td>
</tr>
<tr>
<td>ABC</td>
<td>$\text{FoodNumber} = \frac{N_p}{2}$</td>
<td>Number of food sources</td>
</tr>
<tr>
<td>BBO</td>
<td>$\text{pmutate} = 0$</td>
<td>Mutation probability</td>
</tr>
<tr>
<td></td>
<td>$\text{pmodify} = 1$</td>
<td>Habitat modification probability</td>
</tr>
<tr>
<td></td>
<td>$\text{pmutate} = 0.005$</td>
<td>Initial mutation probability</td>
</tr>
</tbody>
</table>

### Table 12
Comparison of the BSA with DE, PSO, GA, ABC and BBO for solving different OPF problems.

<table>
<thead>
<tr>
<th>Case</th>
<th>BSA</th>
<th>DE</th>
<th>PSO</th>
<th>GA</th>
<th>ABC</th>
<th>BBO</th>
</tr>
</thead>
<tbody>
<tr>
<td>CASE 1</td>
<td>799.0760</td>
<td>799.0376</td>
<td>800.9310</td>
<td>800.1636</td>
<td>799.0541</td>
<td>799.1267</td>
</tr>
<tr>
<td>CASE 2</td>
<td>918.1038</td>
<td>909.0855</td>
<td>963.5922</td>
<td>965.6194</td>
<td>925.3049</td>
<td>917.2192</td>
</tr>
<tr>
<td>CASE 3</td>
<td>1555.8082</td>
<td>1550.3449</td>
<td>1562.0027</td>
<td>1563.8450</td>
<td>1554.8035</td>
<td>1557.0275</td>
</tr>
<tr>
<td>CASE 4</td>
<td>646.1504</td>
<td>645.3627</td>
<td>647.2879</td>
<td>649.9246</td>
<td>650.1636</td>
<td>650.3275</td>
</tr>
<tr>
<td>CASE 5</td>
<td>769.1823</td>
<td>767.7043</td>
<td>820.2658</td>
<td>828.0635</td>
<td>799.7922</td>
<td>807.1644</td>
</tr>
<tr>
<td>CASE 6</td>
<td>1402.8772</td>
<td>1399.6596</td>
<td>1406.3864</td>
<td>1407.0969</td>
<td>1410.0968</td>
<td>1410.4215</td>
</tr>
</tbody>
</table>

In order to evaluate the convergence speed of the BSA, the following approach is proposed in this paper. First, the values of the objective function (average values over all runs) are recorded at four cut-points of the search corresponding to 20%, 40%, 60% and 80% of the maximum number of iterations. Then, the percentage that represents the value of this objective function compared with the final value (average of final values over all runs) reached by the algorithm is calculated. These percentages are reported in Table 13. The final row is the average, calculated on all cases for each cut point. From this table we can say that the BSA is very efficient in solving several OPF problems with different complexities. For the first cut-point, the BSA has reached already more than 85% of the final value of the objective function for all cases except for CASE 16. For cut-point 2, cut-point 3 and cut-point 4 the BSA has reached more than 95%, 97%, and 99% of the final value objective function, respectively.

### Conclusion

In this paper, a newly developed algorithm called the BSA is implemented to solve several OPF problems. Three test systems and sixteen cases have been investigated in order to evaluate the performance of the proposed algorithm. Furthermore, the obtained results have been compared to those obtained using other well-known optimization algorithms such as DE, PSO, ABC, GA and BBO. The main conclusion that can be drawn from this paper is that the BSA is a very effective and robust algorithm for solving OPF problems. It has good convergence characteristics and can achieve
better performances than some well-known optimization algo-
rithms. This is more pronounced when dealing with large scale
power systems. Finally, it is recommended that a multiobjective
BSA algorithm based on Pareto optimal solutions has to be de-
veloped and applied to solve OPF problems in further studies.

Acknowledgments

Dr. M. A. Abido would like to acknowledge the support provided
by King Abdulaziz City for Science and Technology (KACST)
through the Science and Technology Unit at King Fahd University
of Petroleum and Minerals (KFUPM) for funding this work through
project # 14-ENE265-04 as a part of the National Science, Technol-
ogy and Innovation Plan (NSTIP).

References

based optimal power flow using incremental variables. Int J Electr Power
Formulations and deterministic methods; 2012. p. 221–58.
algorithm based on fuzzy multi-objective technique for optimal power flow
security and transient stability constrained optimal power flow. Int J Electr
[7] Bouchekara HREH. Optimal power flow using black-hole-based optimization
[8] Bouchekara HREH, Abido Ma, Boucherma M. Optimal power flow using
teaching-learning-based optimization technique. Electr Power Syst Res
[9] Bouchekara HREH, Abido Ma, Chab aR. Mehanssi R. Optimal power flow using
the league championship algorithm: a case study of the Algerian power
[13] Bhowmik AR, Chaikraborty AK. Solution of optimal power flow using non
-dominated sorting multi objective opposition based gravitational search
Application of imperialist competitive algorithm with its modified
techniques for multi-objective optimal power flow problem: a comparative
study. Inf Sci (Ny); 2014:291:225–47.
[16] Allahsahi M, El-Hawary M. Applications of computational intelligence
techniques for solving the revived optimal power flow problem. Electr
[17] Frank S, Stepnowavice I. Optimal power flow?: a bibliographic survey II
[18] Civicioglu P. Backtracking search optimization rithm for numerical
optimization algorithm (BSA) with other evolutionary algorithms for global
[20] Civicioglu P. Circular antenna array design by using evolutionary search
[21] Shaftullah M, Abido MA, Coelho LS. Design of robust PSS in multimachine
power systems using backtracking search algorithm: 2015.
[22] El-Fergany A. Multi-objective allocation of multi-type distributed generators
along distribution networks using backtracking search algorithm and fuzzy
[23] Hendrix EM, G.-Töth B. Introduction to nonlinear and global optimization,
piscer.cornell.edu/\matpower#docsm.html>.
[28] Abou EI Ela AA, Abido MA, Spea SR. Optimal power flow using differential
[29] Hardiainsyah H. A modified particle swarm optimization technique for
economic load dispatch with valve-point effect. Int J Intell Syst Appl
for optimal power flow considering prohibited zones and valve point effect.
[31] Alisamii J, Syukurul JI, Al-Othman A. A hybrid GA–PS–SQP method to solve
power system valve-point economic dispatch problems. Appl Energy
2010;87:1773–81.
[33] 2014 OPF PROBLEMS. <https://www.uni-du.de/ieee-wgmho/competition
2014n.d>.

Table 13
Convergence speed of the BSA algorithm.

<table>
<thead>
<tr>
<th>Case</th>
<th>Cut-point 1</th>
<th>Cut-point 2</th>
<th>Cut-point 3</th>
<th>Cut-point 4</th>
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<tbody>
<tr>
<td>CASE 1</td>
<td>99.12</td>
<td>99.66</td>
<td>99.86</td>
<td>99.95</td>
</tr>
<tr>
<td>CASE 2</td>
<td>91.81</td>
<td>95.89</td>
<td>98.25</td>
<td>99.38</td>
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<tr>
<td>CASE 3</td>
<td>97.35</td>
<td>99.16</td>
<td>99.76</td>
<td>99.92</td>
</tr>
<tr>
<td>CASE 4</td>
<td>98.19</td>
<td>99.37</td>
<td>99.72</td>
<td>99.89</td>
</tr>
<tr>
<td>CASE 5</td>
<td>89.11</td>
<td>95.34</td>
<td>97.56</td>
<td>99.00</td>
</tr>
<tr>
<td>CASE 6</td>
<td>96.18</td>
<td>98.79</td>
<td>99.63</td>
<td>99.87</td>
</tr>
<tr>
<td>CASE 7</td>
<td>98.07</td>
<td>99.28</td>
<td>99.76</td>
<td>99.91</td>
</tr>
<tr>
<td>CASE 8</td>
<td>91.46</td>
<td>95.55</td>
<td>97.75</td>
<td>99.09</td>
</tr>
<tr>
<td>CASE 9</td>
<td>96.85</td>
<td>98.72</td>
<td>99.45</td>
<td>99.81</td>
</tr>
<tr>
<td>CASE 10</td>
<td>99.46</td>
<td>99.82</td>
<td>99.90</td>
<td>99.96</td>
</tr>
<tr>
<td>CASE 11</td>
<td>93.65</td>
<td>99.70</td>
<td>99.93</td>
<td>99.98</td>
</tr>
<tr>
<td>CASE 12</td>
<td>86.04</td>
<td>96.56</td>
<td>98.81</td>
<td>99.64</td>
</tr>
<tr>
<td>CASE 13</td>
<td>94.98</td>
<td>99.14</td>
<td>99.68</td>
<td>99.89</td>
</tr>
<tr>
<td>CASE 14</td>
<td>92.89</td>
<td>99.72</td>
<td>99.92</td>
<td>99.98</td>
</tr>
<tr>
<td>CASE 15</td>
<td>93.14</td>
<td>99.78</td>
<td>99.93</td>
<td>99.98</td>
</tr>
<tr>
<td>CASE 16</td>
<td>9.96</td>
<td>96.28</td>
<td>98.92</td>
<td>99.74</td>
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<tr>
<td>Average</td>
<td>88.70</td>
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<td>99.30</td>
<td>99.75</td>
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